

# IMPACT OF ROUGHNESS ON OPTIMAL DESIGN OF COMPOSITE CHANNEL

*A thesis submitted*

in partial fulfillment of the requirements

for the Degree of

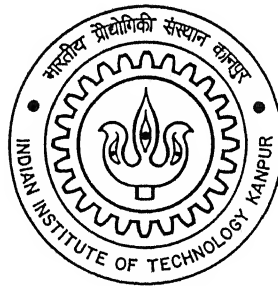
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*in*

**Civil Engineering**

*by*

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*to the*

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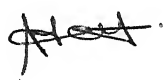
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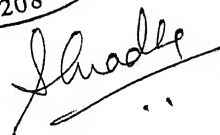
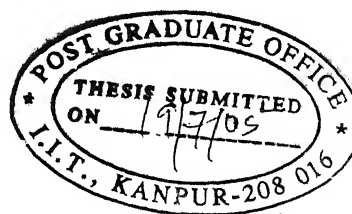
## CERTIFICATE

It is to certify that this thesis entitled **“Impact of Roughness on Optimal Design of Composite Channel”**, by **Satyendra Pratap Singh**, Roll No. Y3103045 has been carried out under my supervision in partial fulfillment of the requirements for the degree of Masters of Technology in Civil Engineering of Indian Institute of Technology Kanpur, India.



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# Abstract

This study investigates the impact of equivalent roughness calculation method on the optimal design of composite channel. Seventeen different methods are investigated. Three methodologies have been used to solve these models that are division of area into sub areas by vertical lines at every places of change in geometry or roughness, division by bisecting lines at every places of change in geometry, and incorporating perimeter along the imaginary line. Techniques of genetic algorithm and classical method (SQP) have been used to solve the optimization problems in all the cases. All these models have been solved for three scenarios having no restriction, restriction on top width, and restriction in side slopes. In this study, a technique has been proposed to determine the roughness along the imaginary vertical lines and this technique has been used in the problem formulation of optimal design of trapezoidal composite channel. The results obtained in this study indicate that the choice of equivalent roughness formula can impact channel design considerably and needs to be chosen carefully giving due consideration to actual conditions in field being closest to the assumption of the method. Also the technique of GA was successfully employed for the purpose of optimal design of composite channel.



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Satyendra Pratap Singh

*Dedicated to my parents*

# Table of Contents

Certificate	ii
Abstract	iii
Acknowledgement	iv
Dedication	v
Table of contents	vi
List of Figures	ix
List of tables	x
<b>1. Introduction</b>	<b>1</b>
1.1 General	1
1.2 Objective of the thesis	4
1.3 Organization of the thesis	5
<b>2. Literature Review</b>	<b>6</b>
2.1 General	6
2.2 Classical methods	7
2.3 Genetic algorithm	11
2.4 Summary	13
<b>3. Genetic Algorithm</b>	<b>14</b>
3.1 Need of Genetic Algorithm	14

3.2 Basics of Genetic Algorithm	15
3.3 Fundamental operations of GA	16
3.3.1 Fitness Function	17
3.3.2 Selection Operator	17
3.3.3 Crossover Operator	18
3.3.4 Mutation Operator	19
<b>4. Model Formulation</b>	<b>22</b>
4.1 Introduction	22
4.2 Problem Formulation	23
4.2.1 Properties of trapezoidal composite channel	23
4.2.2 Uniform flow equation	26
4.2.3 Basic problem formulation	27
4.3 Equations and assumptions of different Models	28
4.4 Scenario II	38
4.5 Scenario III	39
4.6 Solution Methodology	39
4.4.1 First methodology: Sub area division by vertical lines	41
4.4.2 Second Methodology: Sub area division by bisecting lines	41
4.7 Proposed optimization model	42
4.7.1 Approach I: Model Q1	43
4.7.2 Approach I: Model Q2	44
4.7.3 Determination of $K_1$ and $K_2$	45

<b>5. Discussion of Results</b>	<b>49</b>
5.1 General	49
5.2 Results from method-1 scenario I	50
5.3 Results from method-1 scenario II	51
5.4 Results from method-1 scenario III	52
5.5 Results from method-2 scenario I	53
5.6 Results from method-2 scenario II	54
5.7 Results from method-2 scenario III	55
5.8 Results and discussion for model Q1 and Q2	56
<b>6. Summary and Conclusion</b>	<b>71</b>
6.1 Summary	71
6.2 Conclusions	73
6.3 Future scope	74
<b>References</b>	<b>75</b>
<b>Appendix</b>	<b>80</b>

# List of Figures

Figure 3.1: Crossover operator	19
Figure 3.2: Mutation operator	20
Figure 3.3: Flow chart of GA	21
Figure.4.1: Trapezoidal composite channel cross section	24
Figure 4.2: Trapezoidal composite channel with bisecting lines of angle	43
Figure A.1: Area of sub sections	80

# List of Tables

Table 4.1: Different parameters used to formulate optimization problem	40
Table 4.2: Different parameters used to solve optimization problem with GA	40
Table 5.1: Results from various models for vertical line division method, scenario I solved with GA	58
Table 5.2: Results from various models for vertical line division method, scenario II solved with GA	59
Table 5.3: Results from various models for vertical line division method, scenario III solved with GA	60
Table 5.4: Results from various models for vertical line division method, scenario I solved with classical method	61
Table 5.5: Results from various models for vertical line division method, scenario II solved with classical method	62
Table 5.6: Results from various models for vertical line division method, scenario II solved with classical method	63
Table 5.7: Results from various models for bisecting line division method, scenario I solved with GA	64
Table 5.8: Results from various models for bisecting line division method, scenario II solved with GA	65
Table 5.9: Results from various models for bisecting line division method, scenario III solved with GA	66
Table 5.10: Results from various models for bisecting line division method, scenario I solved with classical method	67

Table 5.11: Results from various models for bisecting line division method, scenario II solved with classical method	68
Table 5.12: Results from various models for bisecting line division method, scenario III solved with classical method	69
Table 5.13: Results for Model $Q_1$ and $Q_2$	70



# Chapter 1

## Introduction

### 1.1 General

Manmade channels are major conveyance systems for delivering water for various purposes such as irrigation, water supply, flood control, inland navigation, and rehabilitation, etc. Channel design projects are one of the most expensive civil engineering projects in the world. The total cost of these projects includes the cost of construction and cost of operation, maintenance and repair (OMR). The lengths of channels carrying water for various purposes can be very large (about hundreds of kilometers) so that any savings in the channel projects tends to save a large amount of money and manpower. The main concern in the design of a channel system is to minimize its construction cost to safely carry the design discharge. The total construction cost of channel mainly includes excavation cost and lining cost. Different lining materials for different sides may be needed due to one or more of the following reasons: (a) certain site conditions; (b) limited right of way; (c) availability of lining materials in local areas.

Since different materials may be needed in lining of different sides and bed, the resistance offered to flowing water along sides and bed in such channels may be distinctly different

resulting in different roughness along the wetted perimeter of a channel cross section. These types of channels are called composite channels. In composite channels, the roughness and hence shear stress may be distinctly different in different parts of the channel cross section, hence an equivalent uniform roughness coefficient is needed. The equivalent roughness coefficient equation incorporates flow geometric elements and corresponding roughness coefficient value (Chow, 1959). Traditionally, the equivalent roughness coefficient of a composite channel cross section is conveniently expressed for Manning's roughness ( $n$ ) using weighted sum of the entire resistance coefficients along the perimeter. In computations, usually finite discretization approach is used by dividing the cross-section into number of sub cross-section areas, wetted perimeters, and hydraulic radii and the weighing factor is the function of these parameters (Yen, 2002).

For composite channels, depending upon different assumptions about the relationship of discharge, velocities, forces, and shear stresses between component subsections and total cross section, numbers of formulae has been proposed. Since each formula has its own assumptions and limitations; they are distinctly different from each other. In this study, the impact of different assumptions in calculation of equivalent roughness on optimal design of composite channel has been studied. In the various formulae, weighing factors are calculated by dividing the cross-section into the number of sub-section areas, but the location of imaginary boundary line separating different sub areas is not clear. There are several assumptions for locations of imaginary boundary line given by Yen (2002). Some of these include: (a) vertical lines extending from every break point of the geometry or boundary roughness up to the surface water; (b) bisect lines of every angle at the geometry

or roughness break point; (c) a horizontal or near horizontal line joining the two breaks of the channel at the bank- full stage, separating the compound channel into two parts: the lower main channel section and upper flood channel section; (d) a variation of (c) by further subdividing the lower main channel and upper main channel by bisect angle lines or vertical lines. Since all the previous researchers in the area of channel optimization used method (a) that is the simplest approach for dividing total area into segments, in this study, a detailed examination of the influence of the position of the line of sub area division by applying methods (a) and (b) on the optimal design of the composite channel has been investigated.

Some of the formulae are based on the assumption that there are different velocities in different subsection. When the velocities in the two adjoining areas are very different, then there exists a very high velocity gradient and hence shear stress at the position of imaginary vertical line. Therefore, it is needed to modify the segmental wetted perimeter to take care of the resistance to flow at such boundaries. This has been investigated in this study by incorporating the portion of the imaginary vertical distances corresponding to such boundaries in the segmental wetted perimeters.

The optimization problem for channel design is a constrained optimization problem involving minimization of total construction costs. Manning's equation of uniform flow in the form of an equality constraint is normally employed. Any equality constraint that is nonlinear in nature would make the optimization problem a nonconvex one (Deb 2001). Classical optimization solution procedure may not be suitable to solve such an

optimization problem as they have tendency to converge to a local optima. Since all the classical and gradient based search techniques start search from a single initial solution, there is higher probability to getting stuck into a local optima. Hence, this type of problem must be solved with a global optimization strategy that is based upon: (a) combination of deterministic and probabilistic approaches; and (b) systematic evolution of the solution in the direction of the global improvement, and concept of competitive evolution (Duan et al. 1993). Therefore, evolutionary method of optimization (Genetic Algorithm) is used to solve the problem as it uses probabilistic and deterministic approach starting its search from a set of possible solutions and has a higher probability of obtaining global optimal solution.

## **1.2 Objectives of the thesis**

The objectives of this study are as follows:

1. Analyzing the impact of equivalent roughness coefficient method on the optimal design of composite channel
2. Influence of the position of sub area division on the optimal design of the channel
3. Developing a new optimization program based on the incorporation of the partial wetted perimeter along the imaginary vertical boundaries as area subdivision lines
4. Evaluating various scenarios of optimal design based on the practical situation.

## 1.3 Organization of the thesis

This chapter presents the brief overview of the problem of the optimal design of composite channels. Need of using genetic algorithm to solve the optimization problem of composite channel cross-section design has been also illustrated here. This chapter also presents the need of different scenarios in optimal design of composite channel, influence of line of sub-area division and also accounting the effect of wetted perimeter along the imaginary subdivision on optimal design cost. Chapter 2 consists of literature review of the optimal design of open channel cross section and use of genetic algorithm in past in the closely related areas. Chapter 3 gives the brief overview of binary coded genetic algorithm that has been used in this work. Chapter 4 describes different models, scenarios, approaches, and the solution methodology to solve the optimization problem of composite channel cross section design. Chapter 5 presents the results and their discussion and finally summary, conclusion, and scope for future work have been discussed in chapter 6.

# Chapter 2

## Literature review

### 2.1 General

Channel design projects have been in existence since earlier civilization. The time, when man started agriculture, which is rain dependent and rainfall is highly uncertain, one needed artificial methods to irrigate crops. That time artificial channel designing projects came into existence. There are a lot of evidences that in ancient river valley civilizations (Sindhu River Valley Civilization, 3500-2500 BC) man had designed artificial channels. In ancient literatures like Arthashastra by Kautilya (about 200 BC) one can find methods to design channels. But in modern era, the scenario has been completely changed. Manmade channels are used not only serving for the irrigation purposes; but also for fresh water supply, inland navigation, flood control rehabilitation etc. With the development of sciences and optimization techniques researchers and hydraulic engineers have shown interest in optimizing the channel design problems. Initially, methods of calculus were used to obtain most efficient hydraulic section with least resistance.

## 2.2 Classical methods

In last 25 years, researchers have done a lot of work in the area of optimal channel design. Mironenko et al. (1984) proposed that parabolic shaped unlined and lined canals are hydraulically more stable. The extra stability of the parabolic channels is due to the fact that the only one point of the cross section is at the maximum side slope. A lined parabolic channel has no sharp edges of stress concentration where cracks may occur. Guo and Hughes (1984) designed optimal channel cross-section from the first principle of calculus. They presented an analytical procedure to determine the dimensions of a trapezoidal open channel. They proposed two strategies; the first strategy yields optimal channel shape that provides the least hydraulic resistance and the second one yield least cost channel geometry. Differing from tradition, for the first time they incorporated freeboard as input parameter in optimization problem, as in traditional methods, freeboard was provided after getting optimal dimensions. They presented the results in a graphical form for least hydraulic resistance using a series of figures that can be used in design by practicing engineers. The results revert back to traditional solutions if freeboard parameter is set equal to zero. They did not present any graphical solution for minimum construction cost but proposed a cost function, related to excavation depth and channel width that yields a general expression for a least cost trapezoidal cross-section. Solutions for least cost rectangular channel cross-section with or without freeboard were also demonstrated in their study.

Flynn and Marino (1987) presented the concept of economically optimal section. They constructed canal cross-section designs for uniform flow and compared by non-dimensional shape parameter. They exposed basic relationship among the cross-section shapes and design variables that are wetted perimeter, water depth, water surface width, cross section area, lining volume, and the excavation volume. The relations are used to uncover robust rules that can determine optimal cross-section of canal design for elementary problems directly from flow information such as capacity, velocity, slope and roughness. For problems involving complex limits and economics, the relations are combined with optimization methods to solve for the economically optimal cross section. Loganathan (1991) presented optimality conditions for a parabolic channel cross-section. He accounted for freeboard and put limits on velocity and channel dimensions in his optimization formulation. He stated analytically the necessary conditions for minima. He found that formulation with an objective function consisting of minimization of excavation and lining costs might favor slightly wider channels when the ratio of cost of lining to cost of excavation is high. He has given design aids in the form of tables along with formal procedures to create additional tables, which are necessary for dealing with depth dependent freeboard.

Froehlich (1994) implemented, for the first time, limited top width and depth as additional constraints in cross-section of nonerodible trapezoidal open channel optimization problem that are likely to occur in the field. He solved the constrained minimization problem of finding the best hydraulic channel cross-section using Lagrange multiplier method with the dimensionless forms of the constraints and objective function. Monadjemi (1994) used



Lagrange's method of undetermined multipliers to determine the best hydraulic channel section for a given flow roughness and bed slope. He solved the optimization problem of the channel section by minimizing the wetted perimeter or cross-section subjected to a constraint, which may be any flow equation (i.e. Manning's equation of uniform flow). He applied this method on the standard sections as well as the round bottom triangular sections and found that the minimizations of cross-section area as well as minimization of wetted perimeter are mathematically equivalent. Swamee (1995) investigated explicit equations for the design variables of various canal sections. He found that triangular and rectangular sections are equally optimal in the sense that they yield the same minimum area and semicircular section has least flow area and flow perimeter. Thus, constrained by straight line geometry, the optimal canal section becomes closest to the semi-circular section. Swamee et al. (2000) investigated the optimization problem of open channel cross-section design considering minimum seepage loss. They applied nonlinear optimization technique to solve the explicit equations for design of minimum seepage loss channel section that are obtained by using seepage loss equation and uniform flow equation.

As lining materials is one of the basic constituents in the channel design cost, a lot of attention is needed to select the lining material for the whole wetted perimeter. There may be several site conditions, for that it becomes necessary to use different lining materials on different sides and bed. Cost of lining materials mainly depends upon their availability in local area otherwise transportation cost will increase the whole construction cost. Attention on this phase of the problem was given by Trout (1982) who developed direct

algebraic techniques to determine open channel cross-section design which minimizes lining material cost when different materials are used for lining bed and sides. He presented solutions in the form of graphs, which indicates optimal parameter combination, and cost of deviation from the optimal design. He considered only minimization of the lining cost of the trapezoidal channel but did not consider minimization of the cross-sectional area. Das (2000) formulated optimal design problem in nonlinear optimization framework with cost per unit length of composite channel as objective function. Manning's equation, positive values of design variables and specified values of side slopes or top width were used as constraint. He considered the excavation cost and lining cost as objective function. He used Lagrange Multiplier method to convert the constraint optimization problem to unconstrained optimization problem and developed a computational methodology using first order necessary condition for optima to solve this problem. Recently, Jain et al (2004) solved the problem of composite channel design using genetic algorithm for first time for different scenarios including top width constraint, side slope constraint and velocity constraint.

There are many equations that can be used to calculate the equivalent roughness of composite channels. Yen (2002) presented a list of many of these equations that can be used to formulate different models for composite channel optimization problem. There are several inherent assumptions and limitations to each equation of equivalent roughness that are also applied to optimization models formulated using these equations. The final results may be different for different models because of these different equations and their assumptions. In the previous approaches, Das (2000) used Horton's (1933) equation for

calculation of equivalent roughness which is a lumped approach and considers equal velocity in all segmental areas of the composite channel that is equal to mean velocity of the channel. Some other methods of calculation of equivalent roughness also assume same approach in the sense that there is a constant velocity across the cross-section but few of them allowed the spatial variation across the cross section. It should be realized that the different roughness coefficient across the channel cross section should modify the velocity distribution. This drawback in the Horton's method can be overcome by using Lotter's (1933) approach of calculating equivalent roughness in composite channels. This method does not assume segmental velocities as equal to mean velocity and allows spatial variations in segmental velocities across the cross section. Jain et al (2004) proposed a new model that was used in optimization problem considering the spatial variation across the cross section.

## 2.3 Genetic Algorithm

In last decade, the researchers working in the area of hydrology and water resources have shown interest in using GA to solve optimization problem. Wang (1991) successfully applied GA in calibration of conceptual rainfall runoff model for data from a particular catchment. There were seven parameters in his model and all seven parameters have been optimized by using GA. Cieniaveski et al. (1995) used GA to solve multi-objective ground water monitoring problem and compared the results with the results from simulated annealing. They found that GA is able to find a larger number of non dominated points on the trade off curve while simulated annealing found fewer points. Liang et al. (1995) used

GA in search for optimal values of catchment's calibration parameters. They linked GA to a widely used catchment model, the storm water management model (SWMM) and applied to a catchment in Singapore. They considered six storms and out of them three were used for calibration and three were used for verification of the results. Their study showed that GA required only a small number of catchment model simulations and yet yielded relatively high peak flow prediction accuracy. Gentry et al. (2001) employed GA to determine areas contaminating a semi confined aquifer. There are many other researchers who used GA successfully in different areas of water resources such as urban drainage modeling (Rouch et al., 1999); optimal design and rehabilitation of water distribution system (Wu, 2001); estimation of aquifer parameters (Samuel, 2003); location of aquifer leakage areas (Gantry, 2003). Jain et al. (2004b) used real coded GA to train ANN rainfall runoff model and Jain et al. (2005) used real coded GA to determine unit pulse response function using historical data from watershed. There are many other works that have been carried out by different researchers showing that GA can be efficiently used in the complex problems of water resources. Optimal channel designing problems are not far away from the reach of GAs. Recently, Jain et al (2004) solved the problem composite open channel design using genetic algorithm for first time for different scenarios including top width constraint, side slope constraint and velocity constraint. They also proposed a model based on the distributed approach, which allows spatial variation in velocity across a composite channel. They presented results in form of optimal design graphs to determine dimensions of optimal trapezoidal composite channel section.

## 2.4 Summary

Channel designing projects started in very old ages but the main concern in this area was given in last two decades. Initially, optimal design was carried out with the methods of calculus to obtain most efficient section with least resistance. The techniques employed to solve the problem of optimal channel design mainly has been classical method; however, a few researches have used GAs for this purpose. The channel optimization problem involves minimization of total construction cost with uniform flow equations and non negativity constraints. Various scenarios involving additional constraints have been attempted. Recently, some researchers have proposed solutions to the optimal design of composite channels. This study extends the existing work in the area of optimal design of composite channels.

# Chapter 3

## Genetic Algorithm

### 3.1 Need of Genetic Algorithm

The need for solving optimization problems arises in almost every field and, in particular, is a dominant problem in water resources systems. Consequently, many analytical and numerical optimization techniques have been developed. There exist a great number of functions, that are discontinuous, nondifferentiable, nonconvex, and multimodal, which are beyond analytical methods and pose profound difficulties for traditional numerical techniques. Generally, traditional optimization problem solution techniques are based on a deterministic relationship between the model parameters and its performance but now a days these techniques have been unable to optimize such complex problems. (McKinney and Lin 1994, Reed et al. 2000). In addition to these, there are some common difficulties with most of the classical direct and gradient based techniques; (a) convergence to an optimal solution depends on the chosen initial solution; (b) most algorithms tend to get stuck in a sub optimal solution; (c) an algorithm efficient in solving one optimization problem may not be efficient in solving a different optimization problem; (d) algorithms are not efficient in handling problems having a discrete search space; and (e) if search space contains some local optima, classical methods get attracted to any of locally optimal

solution, and there is no escape. Therefore, new and more robust optimization techniques, which are capable of handling such problems, are needed. The search for more efficient numerical optimization methods has led researchers to reproduce some mechanisms that are naturally robust such as genetic algorithms (GAs).

GA is a search technique based on the concept of natural selection inherent in the natural genetics, which combines an artificial survival of the fittest with genetic operators abstracted from nature (Holland 1975). The major difference between GA and other gradient based or traditional optimization solution techniques is that GA start search with a population of many possible solutions; whereas other classical optimization solution techniques start their search with a single point. Hence, GAs are less susceptible to getting “stuck” at local optima than gradient search or other traditional classical optimization methods, rather they search for global optima. In the past decade, GAs have been applied successfully to a number of problems such as optimizing simulation models, fitting nonlinear curves to data, solving systems of nonlinear equations, and machine learning (Deb, 2001).

### **3.2 Basics of Genetic Algorithm**

First of all, an initial population of solutions is generated using a random generator. Every population is presented by a set of parameter values that describe the problem. Traditionally, GAs have been developed by using binary coding, in which a chromosome or string is represented by a string of binary bits i.e., 0's and 1's which are called genes

that can encode integers, real numbers, or anything else appropriate to the problem. Binary strings are easy to operate on, and within any gene, binary representations can be mapped to values in the range feasible for the variable represented (Goldberg, 1989). In this study, the parameters or decision variables are encoded as substrings of binary digits having a specific length. The length of the substring is determined according to the desired solution accuracy, and is dependent on the specified range of a parameter i.e., upper and lower bounds and the precision requirement of the parameter. By increasing the substring length, more accuracy in the solution can be achieved. Any precision for variables can be achieved by changing the string length and lower and upper bounds.

$$x_i = x_i^{\min} + \frac{x_i^{\max} - x_i^{\min}}{2^{l_i} - 1} DV(s_i) \quad (3.1)$$

Where  $l_i$  is length used to code the  $i$ -th variable and  $DV(s_i)$  is the decoded value of the string  $s_i$ ,  $x_i$  is actual value of the variable,  $x_i^{\min}$  and  $x_i^{\max}$  are lower and upper bounds for the variable and the complete string will be  $s = \bigcup_{i=1}^n s_i$ . The above mapping function represented by equation (3.1) allows any arbitrary precision for a variable and different precision for different variables, by changing the length of substring.

### 3.3 Fundamental Operations of GA

GA starts with randomly generating an initial population of possible solutions. For most GAs these candidate solutions are represented by chromosomes coded using a binary number system. The GA that employs binary strings as its chromosomes is called binary



coded GA. These chromosomes are evaluated based upon their fitness function, in terms of certain objective function. The binary coded GA consist of three basic operators, selection operator, crossover operator and mutation operator. These are discussed in the following sections.

### **3.3.1 Fitness Function**

In GA, solution to the problem is evaluated using fitness functions, also known as objective function. The evaluation of function determines the cost of the solution, which includes different costs involved in the objective function. Based upon the results of such functions, evolutionary procedure may be applied to a set of solutions. In case objective of optimization is maximization of the objective function, a solution with larger fitness value compared to other solutions is better but if the objective of optimization is minimization of the objective function, a solution with smaller fitness value is better as compared to other solutions.

### **3.3.2 Selection Operator**

The objective of selection operator is to make duplicates of good solutions and eliminate bad solutions in the population while keeping the population size constant. There are a number of methods like tournament selection, proportionate selection, and ranking selection by which this task can be achieved, which include identification of good solutions in population, production of multiple copies of good solutions, and elimination of

bad solutions from the population so that bad solutions can be replaced by multiple copies of good solutions. These can be found in Deb (2001).

### 3.3.3 Crossover Operator

Crossover and mutation are driving force in any evolutionary algorithm. The theory behind the crossover operation is that by exchanging important building blocks between two strings that perform well, the GA attempts to create new strings that preserve the best material from two parent strings. Building blocks are the chromosomes that contain information in the form of genes. The number of strings in which material is exchanged is controlled by crossover probability. Crossover operator is a recombination operator that picks up two chromosomes, called parent strings, in mating pool. Crossover location is selected and the genetic information is exchanged between the parent chromosomes with a certain probability ( $p_c$ ) to create two new chromosomes, called child strings. Crossover probability can lie between (0-1.0) and is generally taken from range (0.6 –1.0). This procedure is repeated to other pairs of chromosomes to give a new generation of child population having better fitness value than that of the parents. Crossover has a wide range of possible types, viz., one-point, multiple-point, uniform, intermediate arithmetical, and extended arithmetical which can be found in Deb (2001). The functioning of a single point crossover is explained in figure (3.1) in which  $P_1$  and  $P_2$  represent the parent chromosome and  $C_1$  and  $C_2$  represent the two children. The Character “{” represents the location of single point crossover in both parents.

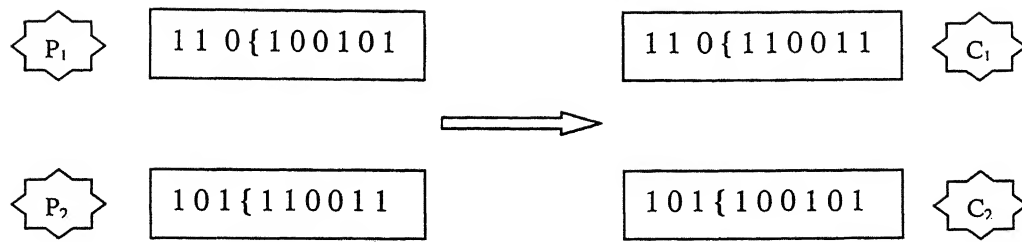
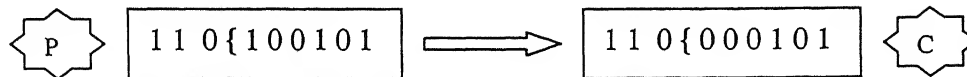


Figure 3.1: Crossover operator

### 3.3.4 Mutation Operator

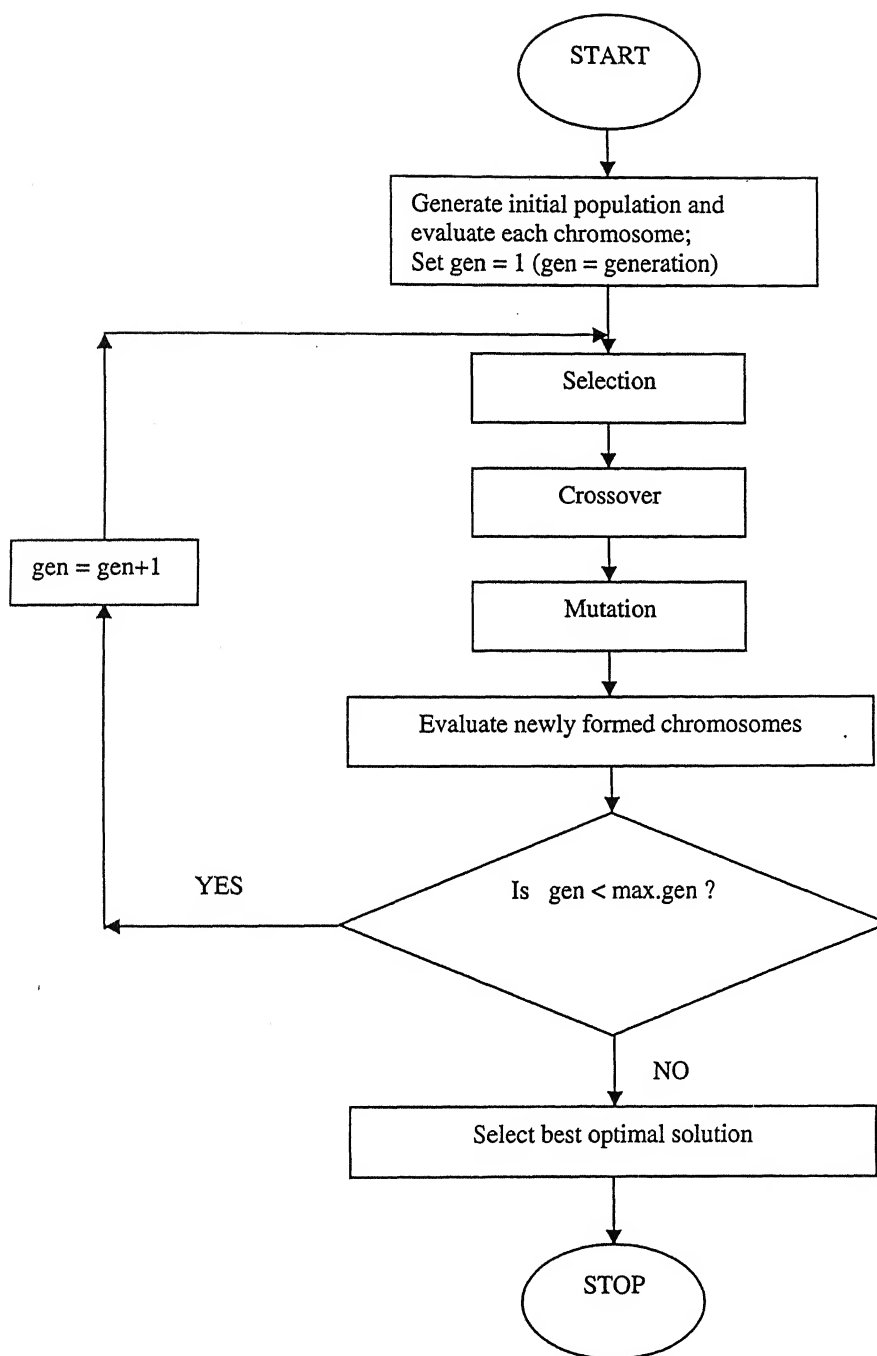
GA is a very aggressive search technique and discards the chromosomes having poor fitness values which may cause our search to converge to local optima. To eliminate this possibility and get global optima, mutation operator is used. Mutation provides a small amount of random search, and helps to ensure that no point in the search space has a zero probability of being examined. The mutation operator generally facilitates the child population to converge towards optimal solution even if the initial population is far from optima. The bit wise mutation operator in binary coded string changes a 1 to a 0 and vice versa with a small mutation probability of  $p_m$ . It is used to maintain the diversity among the child population from one generation to another generation. Proper selection of mutation probability is important for proper working of GA. In previous studies, it has been suggested that a mutation probability inversely proportional to the population size is enough to prevent the search to converge into a local optima.

Figure (3.2) describes the process involved in mutation. “P” represent the parent chromosome and “C” represent the child chromosome. Character “{” represent the location of mutation where bit 1 changes to 0. Mutation probability can lie between (0-1.0) but generally taken very small.



**Figure 3.2: Mutation operator**

The process of selection, crossover and mutation is repeated from one generation to the next, achieving better fitness in each generation. The search is terminated using certain stopping criteria in terms of the convergences of the solution. Figure (3.3) shows the flowchart for a simple GA.



**Figure 3.3: Flow chart of GA**

## Chapter 4

# Model Development

### 4.1 Introduction

In this study seventeen nonlinear optimization problems for optimal design of trapezoidal composite channel cross section have been investigated. All these problems are constrained minimization problem and consist of same objective function which is cost of construction of the trapezoidal composite channel. Uniform flow conditions are ensured using Manning's equation as a constraint. The total cost of construction includes cost of excavation that depends upon cross sectional area, and cost of lining that depends upon the whole wetted perimeter. In this study, attempt has been made to design composite channel in which different lining materials have been used on different sides and bed and having different costs per unit length. Basic differences in all these optimization problems are that they are subjected to different approaches of calculation of equivalent roughness. Since roughness is changing along the wetted perimeter and is distinctly different in different parts; an equivalent roughness ( $n_c$ ) is required to be used in Manning's uniform flow formula (Chow 1959). There are various methods of calculation of equivalent roughness that have been developed by various researchers in the past. The basic differences in these methods are that they have been developed based upon certain assumptions and all

methods have their own limitations. When these equations are used in optimization problem, their assumptions and limitations are liable to modify the entire search space of the problem. This study investigates the impact of roughness on the optimal design of composite channel. The basic formulations of all these optimization problems are as follows:

## 4.2 Problem Formulation

The objective function is to minimize cost function, which is cost of construction per unit length of channel, consist of excavation cost and lining cost and subjected to uniform flow condition and non-negative parameters of design. Before discussing the problem formulation in detail let us define the properties of trapezoidal channel cross section that will be used in problem formulation.

### 4.2.1 Properties of Composite Trapezoidal Channel

Figure 4.1 shows a composite trapezoidal channel cross section. It has base width  $b$ , depth of water  $y$ , freeboard  $f$  and side slopes  $z_1:1$  (Horizontal: Vertical) and  $z_2:1$  (Horizontal: Vertical). Let  $T_w$  be the flow top width and  $T_f$  be the total top width of the cross section. The case when vertical lines are imaginary lines that divide the total area into sub areas as in figure 4.1 let  $A_{w1}$ ,  $A_{w2}$  and  $A_{w3}$  be the wetted flow subsection areas and  $A_w$  be the total wetted flow area. Similarly, let  $P_w$  be the flow wetted perimeter of the cross section and

$P_{w1}$ ,  $P_{w2}$  and  $P_{w3}$  be the wetted lengths on each side. The hydraulic parameters can be calculated using the following equations:

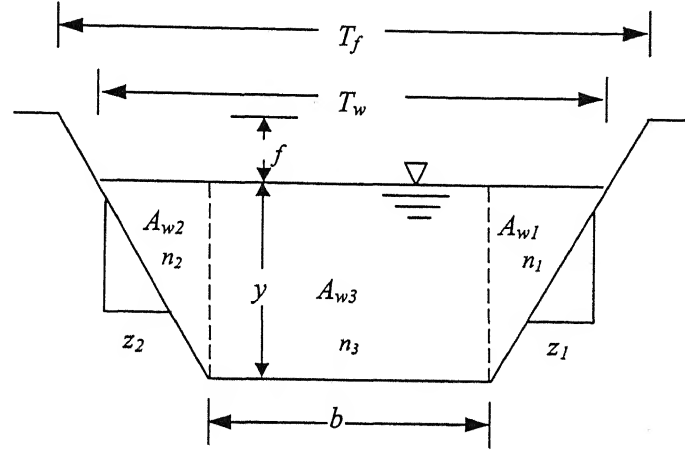


Figure 4.1: Trapezoidal composite channel section

$$A_{w1} = \frac{z_1 y^2}{2} \quad (4.1)$$

$$A_{w2} = \frac{z_2 y^2}{2} \quad (4.2)$$

$$A_{w3} = b y \quad (4.3)$$

$$A_w = b y + (z_1 + z_2) \frac{y^2}{2} \quad (4.4)$$

$$P_{w1} = \left( \sqrt{z_1^2 + 1} \right) y \quad (4.5)$$

$$P_{w2} = \left( \sqrt{z_2^2 + 1} \right) y \quad (4.6)$$

$$P_{w3} = b \quad (4.7)$$

$$P_w = \left( \sqrt{z_1^2 + 1} + \sqrt{z_2^2 + 1} \right) y + b \quad (4.8)$$



Similarly, subsection areas  $A_1$ ,  $A_2$ , and  $A_3$ , the total area  $A_t$ , total wetted perimeter  $P_t$  and the total channel cross section top width  $T_t$  of the trapezoidal channel cross section accounting for freeboard can be written as follows:

$$A_1 = \frac{z_1 (y+f)^2}{2} \quad (4.9)$$

$$A_2 = \frac{z_2 (y+f)^2}{2} \quad (4.10)$$

$$A_3 = b(y+f) \quad (4.11)$$

$$A_t = b(y+f) + (z_1 + z_2) \frac{(y+f)^2}{2} \quad (4.12)$$

$$P_t = \left( \sqrt{z_1^2 + 1} + \sqrt{z_2^2 + 1} \right) (y+f) + b \quad (4.13)$$

$$T_t = b + (z_1 + z_2)(y+f) \quad (4.14)$$

Let,  $P_1$  and  $P_2$  are the length of sides including the freeboard and  $P_3$  be the wetted bed width of the channel. These can be written as:

$$P_1 = \left( \sqrt{z_1^2 + 1} \right) (y+f) \quad (4.15)$$

$$P_2 = \left( \sqrt{z_2^2 + 1} \right) (y+f) \quad (4.16)$$

$$P_3 = b \quad (4.17)$$

But in case when the bisecting lines are dividing the total area into sub areas as in figure 4.2, subsection areas  $A_{w1}$ ,  $A_{w2}$ , and  $A_{w3}$  becomes modified and their equations will be

$$A_{w1} = \left( \sqrt{z_1^2 + 1} \right) \frac{y^2}{2} \quad (4.18)$$

$$A_{w2} = \left( \sqrt{z_2^2 + 1} \right) \frac{y^2}{2} \quad (4.19)$$

$$A_{w3} = b y + (z_1 + z_2) \frac{y^2}{2} - \left( \sqrt{z_1^2 + 1} + \sqrt{z_2^2 + 1} \right) \frac{y^2}{2} \quad (4.20)$$

Method of calculation of these areas is given in Appendix A.

#### 4.2.2 Uniform Flow Equation

The Manning's equation of uniform flow in a channel can be expressed as:

$$\frac{Q n}{\sqrt{S_0}} = \frac{A^{5/3}}{P^{2/3}} \quad (4.21)$$

Where  $Q$  is the total discharge in the channel ( $\text{m}^3/\text{s}$ ),  $S_0$  is the bed slope of the channel,  $n$  is the roughness coefficient of the channel cross section,  $A$  is the channel cross sectional area ( $\text{m}^2$ ) and  $P$  is the total wetted perimeter (m).

### 4.2.3 Basic Problem Formulation

The optimization problem can be represented by the following equations:

$$\text{Minimize: } C(b, z_1, z_2, y) = c_1 A_t + c_2 P_1 + c_3 P_2 + c_4 P_3 \quad (4.22)$$

$$\text{Subjected to: } \Phi(b, y, z_1, z_2) = \frac{Q n_c}{\sqrt{S_0}} = \frac{A^{5/3}}{P^{2/3}} \quad (4.23)$$

$$b \geq 0; \quad z_1 \geq 0; \quad z_2 \geq 0; \quad y \geq 0$$

Where  $A_t$  is the total cross sectional area that is area of excavation as in equation (4.12),  $P_1$ ,  $P_2$  and  $P_3$  are length of sides including freeboard and defined in equations (4.15), (4.16) and (4.17), respectively,  $c_1$  is cost of excavation per sq. meter,  $c_2$  is cost of lining per meter length of perimeter  $P_1$ ,  $c_3$  is cost of lining per meter length of perimeter  $P_2$ ,  $c_4$  is cost of lining per meter length of perimeter  $P_3$ ,  $n_1$ ,  $n_2$  and  $n_3$  are Manning's coefficients for different lining materials used on different sides of perimeter and  $n_c$  is equivalent roughness.

Since the flow equation makes an equality constraint and while solving with GA it becomes a problem because GA has its inherent limitation of not being able to handle equality constraints; therefore it becomes necessary to convert the equality constraint into inequality constraint. It was achieved by introducing a dummy variable  $\varepsilon$  in the optimization problem formulation. Hence, the constraint function will be

$$\Phi(b, y, z_1, z_2) = \varepsilon - \left| \frac{Q n_c}{\sqrt{S_0}} - \frac{A^{5/3}}{P^{2/3}} \right| \geq 0 \quad (4.24)$$

### 4.3 Equations and Assumptions of Different Models

Seventeen models based upon methods of calculation of equivalent roughness have been formulated. Basic formulations of all these optimization models are as given above. Objective functions of all these models are exactly same (equation 4.22) but the basic difference is methods of calculation of equivalent roughness that makes modifications in search space. Methods of calculation of equivalent roughness and their inbuilt assumptions and limitations are as follows:

#### Model A

This method of calculation of equivalent roughness was given by US army corps of engineers (USCOE), Los Angeles (1973). In this method equivalent roughness is calculated by summing component  $n$  weighted by area ratio. The shear assumption involved in this method is that the shear velocity is weighted sum of sub area velocity i.e

$$\sqrt{gRS} = \sum \left( \frac{P_i}{P} \sqrt{gR_i S_i} \right) \quad (4.25)$$

$$\text{and } \left( \frac{V_i}{V} \right) = \left( \frac{R_i}{R} \right)^{7/6} \quad (4.26)$$

Equivalent roughness in this method will be as follows:

$$n_c = \sum \frac{n_i A_i}{A} \quad (4.27)$$

### Model B

In this method, equivalent roughness is calculated by taking square root of sum of the square of the component  $n$  weighted by sub area ratio. Assumption involve in this model is total resistance force is equal to sum of sub area resistance forces i.e.

$$P \gamma R S = \sum P_i \gamma R_i S_i \quad (4.28)$$

$$\text{and } \left( \frac{V_i}{V} \right) = \left( \frac{R_i}{R} \right)^{2/3} \quad (4.29)$$

Equivalent roughness in this method will be as follows:

$$n_c = \sqrt{\sum n_i^2 \frac{A_i}{A}} \quad (4.30)$$

### Model C

In this method equivalent roughness is calculated by dividing total area by sum of sub areas normalized by component equivalent roughness. Assumptions involved in this method are that total discharge is sum of sub area discharges i.e.

$$Q = V A = \sum (V_i A_i) = \sum Q_i \quad (4.31)$$

$$\text{and } \left( \frac{S_i}{S} \right) = \left( \frac{R}{R_i} \right)^{4/3} \quad (4.32)$$

Method of calculation of equivalent roughness in this method will be as follows:

$$n_c = \frac{A}{\sum (A_i / n_i)} \quad (4.33)$$

### Model D

This method of calculation of equivalent roughness was given by Colebatch (1941). In this method also sub area ratio is used as weighing factor for component  $n$  to calculate the equivalent roughness. Assumptions in this method are that the total cross sectional mean velocity is equal to sub area velocity i.e.

$$V = V_i \quad (4.34)$$

and energy grade line for total cross sectional area is parallel to that of sub section areas i.e.

$$S = S_i \quad (4.35)$$

Method of calculation of equivalent roughness in this method will be

$$n_c = \left( \frac{\sum (n_i^{3/2} A_i)}{A} \right)^{2/3} \quad (4.36)$$

### Model E

This method of calculation of equivalent roughness was given by Horton (1933). In the above four methods (A to D) component  $n$  were weighted by sub area ratios to calculate the equivalent roughness but in this method sub area wetted perimeter ratio is used as weighing factor to calculate the equivalent roughness. Assumptions in this method are same as above in model D as in equations (4.34) and (4.35). Method of calculation of equivalent roughness in this method will be as:

$$n_c = \left( \frac{1}{P} \sum (n_i^{3/2} P_i) \right)^{2/3} \quad (4.37)$$

### Model F

This method of calculation of equivalent roughness was given by Felkel (1960). In this method of calculation of equivalent roughness the total flow area wetted perimeter is divided by sub area wetted perimeters normalized by component  $n$ . Assumptions of this method are that the total discharge is some of sub area discharges as in equation (4.31) and

$$\left( \frac{S_i}{S} \right) = \left( \frac{R}{R_i} \right)^{10/3} \quad (4.38)$$

Method of calculation of equivalent roughness in this method will be as follows:

$$n_c = \frac{P}{\sum (P_i / n_i)} \quad (4.39)$$

### Model G

This method was given by Pavlovaskii (1931). In this method equivalent roughness is calculated by taking square root of sum of the square of the component  $n$  weighted by sub area wetted perimeter ratios. Assumption involve in this model are as follows:

Total resistance force is equal to sum of sub area resistance forces as in equation (4.38)

$$\text{and } (V_i / V) = (R_i / R)^{1/6} \quad (4.40)$$

Method of calculation of equivalent roughness in this method is

$$n_c = \left( \frac{1}{P} \sum (n_i^2 P_i) \right)^{1/2} \quad (4.41)$$

### Model H

This method of calculation was given by Yen (1991). In this method equivalent roughness the calculated by summing the component  $n$  weighted by sub area wetted perimeter ratios. Assumptions involved in this method are that the total shear velocity is wetted sum of sub area shear velocities or contributing component roughness is linearly proportional to wetted perimeter as in equation (4.25) and the velocity ratio is related to hydraulic radii with the equation (4.40). In this method equivalent roughness will be



$$n_c = \frac{\sum (n_i P)_i}{P} \quad (4.42)$$

In the models A to model D sub area ratios are used as weighing factor and from models E to H sub area wetted perimeter ratio is used as weighing factor for component  $n$  to calculate the equivalent roughness. From model I onwards sub area ratio and wetted perimeter ratio both are used as weighing factor. Here, another parameter called hydraulic radius that is area divided by wetted perimeter is also used.

### Model I

In this method of calculation of equivalent roughness square root of summation of square of component  $n$  weighted by sub area wetted perimeter ratio and also hydraulic radii ratios is taken. Assumptions involved in this method are that the Total resistance force,  $F$ , is some of sub area resistance forces,  $\sum F_i$  as in equation (4.28) and total cross sectional velocity is equal to sub area velocities as in equation (4.34). In this method equivalent roughness will be

$$n_c = \left( \frac{R^{1/3}}{P} \sum \frac{n_i^2 P_i}{R_i^{1/3}} \right)^{1/2} \quad (4.43)$$

## Model J

In this method also equivalent roughness is calculated by taking square root of summation of squares of component  $n$  weighted by sub area wetted perimeter ratios and hydraulic radius ratios in a different way. Assumptions of this method are that the total resistance force is some of sub area resistance forces as in equation (4.28)

$$\text{and } (V_i / V) = (R_i / R)^{1/2} \quad (4.44)$$

Method of calculation of equivalent roughness is as follows:

$$n_c = \left( \frac{\sum n_i^2 P_i R_i^{2/3}}{P R^{2/3}} \right)^{1/2} \quad (4.45)$$

## Model K

This is another method of calculation of equivalent roughness. This method allows spatial variation in velocity across the cross section. It has assumptions that the total discharge is equal to sum of sub area discharges as in equation (4.31) and also sub area grade lines are inversely proportional hydraulic radius

$$(S_i / S) = (R / R_i) \quad (4.46)$$

Equivalent roughness in this method is calculated as:

$$n_c = \frac{PR^{7/6}}{\sum \frac{P_i}{n_i} R_i^{7/6}} \quad (4.47)$$

## Model L

This method was given by Lotter (1933). This method also allows the spatial variation of velocity across the total cross section. Assumptions of this method are that the total discharge is equal to sum of sub area discharges as in equation (4.31) and energy grade lines for sub areas and total cross sectional areas are parallel as in equation (4.35). Equivalent roughness in this method will be

$$n_c = \frac{PR^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}} \quad (4.48)$$

## Model M

This method was given by Ida (1960). In this method of calculation of equivalent roughness the assumptions are, the total discharge is equal to sum of sub area discharges as in equation (4.31), and energy grade lines for sub areas and total cross sectional areas are parallel as in equation (4.35) and also

$$\frac{PR^{5/3}}{\sum P_i R_i^{5/3}} = \frac{AR^{2/3}}{\sum A_i R_i^{2/3}} = 1 \quad (4.49)$$

Equivalent roughness in this method will be

$$n_c = \frac{\sum P_i R_i^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}} \quad (4.50)$$

### Model N

This method was given by Yen (1991). This method does not allow the spatial variation in the velocity across the cross section. The assumptions involved in this method of calculation of equivalent roughness are that the total shear velocity is weighted sum of sub area shear velocities as in equation (4.25) and velocity in total cross sectional area is equal to velocity in sub areas as in equation (4.34). Equivalent roughness in this method will be as:

$$n_c = \frac{\sum (n_i P_i / R_i^{1/6})}{P / R^{1/6}} \quad (4.51)$$

### Model O

This is another method of calculation of equivalent roughness. Assumptions involved in this method are, total shear velocity is equal to sum of sub area shear velocity as in

equation (4.25) and velocity is related to hydraulic radius as in equation (4.29). Equivalent roughness in this method will be

$$n_c = \frac{\sum (n_i P_i R_i^{1/2})}{PR^{1/2}} \quad (4.52)$$

### Model P

This method of calculation of equivalent roughness was given by Yen (1991). This method of calculation of equivalent roughness is based upon some assumptions that are total shear velocity is equal to some of sub area shear velocities as in equation (4.25) and velocity is related to hydraulic radius as in equation (4.44). Equivalent roughness in this method will be:

$$n_c = \frac{\sum n_i P_i R_i^{1/3}}{PR^{1/3}} \quad (4.53)$$

#### 4.3.17 Model Q

Model Q in this study is the one developed by Jain et al. (2004). They proposed a new model formulation for optimal design of composite channel. They used the same objective function that is used in this study but proposed a new constraint which is based upon the spatial variation of velocity across the cross section and used the bisection method for area

sub division. The constraint function for the optimal design of composite channel employed was:

$$\Phi_1(b, y, z_1, z_2) = \varepsilon - \left| \frac{Q}{\sqrt{S_0}} - \frac{A_{w1}^{5/3}}{n_1 \left\{ \left( \sqrt{z_1^2 + 1} \right) y \right\}^{2/3}} - \frac{A_{w2}^{5/3}}{n_2 \left\{ \left( \sqrt{z_2^2 + 1} \right) y \right\}^{2/3}} - \frac{A_{w3}^{5/3}}{n_3 b^{2/3}} \right| \geq 0 \quad (4.54)$$

All of the above problem formulations are called scenario-I in this study in which there are no additional constraints. Some times, it becomes necessary to impose additional constraints in the optimization problem to simulate certain site conditions. Two additional scenarios were investigated for all of the above models. These scenarios as follows:

#### 4.4 Scenario II

Sometimes for practicing engineers, it becomes a problem that there is limited width available to construct the channel. In order to simulate such type of conditions when the top width is limited and restricted up to a maximum value  $T_{max}$ , as used by Froehlich (1994), Das (2000) and Jain et al. (2004), an additional constrain can be imposed on the optimization problem. The additional constraint to the optimization problem will be as follows:

$$\Phi_2(b, y, z_1, z_2) = T_{max} - T_t \geq 0 \quad (4.55)$$

Where  $\Phi_2$  is an additional inequality constraint function limiting the total top width of composite channel to  $T_{max}$ .

### 4.5 Scenario III

Sometimes side slopes becomes a problem for practicing engineers because all soils have some stability criteria that should be satisfied. For the case when it becomes necessary to restrict side slope up to a minimum value and a maximum value due to certain reasons as slope stability criteria, or limited right of way, etc, additional constraints can be imposed on the optimization problem. The additional constraints to the optimization problem will be as follows:

$$\Phi_3(z_1) = z_1 - z_{\min} \geq 0 \quad (4.56)$$

$$\Phi_4(z_2) = z_2 - z_{\min} \geq 0 \quad (4.57)$$

$$\Phi_5(z_1) = z_{\max} - z_1 \geq 0 \quad (4.58)$$

$$\Phi_6(z_2) = z_{\max} - z_2 \geq 0 \quad (4.59)$$

### 4.6 Solution Methodology

In this study seventeen different models (model A to model Q) have been formulated for optimal design of trapezoidal composite channel. Here, all these models differ from each other depending upon the method of calculation of equivalent roughness. All the problems and scenarios have been solved using two different methods: (a) binary coded genetic

algorithm (GA); and (b) sequential quadratic programming (SQP) from MATLAB-7.01 optimization toolbox. Results are presented in form of tables in the next chapter. All the models have three scenarios, namely no restriction (scenario-I), restriction on top width (scenario-II), and restriction on side slopes (Scenario-III). The values of various defined parameters in this study are presented in Table (4.1). These values of various parameters were chosen for comparison purposes because these have been used by the other researchers in the past.

**Table 4.1 Different parameters used to formulate optimization problem**

Discharge ( $Q$ ) $\text{m}^3/\text{s}$	Freeboard (meter)	Slope	$c_1$	$c_2$	$c_3$	$c_4$	$n_1$	$n_2$	$n_3$	$T_{max}$ (m)
100	0.5	0.0016	0.6	0.2	0.25	0.3	0.02	0.018	0.015	7.5

The flowchart of working of GA has been presented in Figure 3.3. While using GA to solve these problems, we require some user defined parameters. The values of different parameters used to solve these problems are listed in Table 4.2.

**Table 4.2 Different parameters used to solve optimization problem with GA.**

Population size	Crossover probability	Mutation probability	Bit length used for different variables	Total bit length	Number of runs
200	0.876	0.0125	20	80	20



#### **4.61 First Methodology: Sub area division by vertical lines**

Almost all the researches of the past relating to optimization problem of trapezoidal open channel cross section are based upon formulation of problem by the simplest approach of division of total cross sectional area into sub areas, that is vertical lines extending from every break point of the geometry or boundary roughness up to the water surface as shown in Figure 4.1. This methodology has been applied to all the models (model A to model Q) for all three scenarios and results by genetic algorithm are presented in the Tables 5.1 to 5.3 for Scenario-I, Scenario-II Scenario-III respectively. Results by SQP (MATLAB 7.01) are presented in Tables 5.4 to 5.6 for Scenario-I, Scenario-II, and scenario-III respectively.

#### **4.6.2 Second Methodology: sub area division by bisecting lines**

As almost in all the models (excluding models E to model H) equivalent roughness depends upon the sub areas to total area ratios, method of division of total area into sub areas modifies this ratio and hence the equivalent roughness coefficient. Since the method of division of total area into sub areas modifies the equivalent roughness coefficient  $n_c$ , it causes modification in the search space of the optimization problem and hence the optimization problem will be different. In this study, another approach that is dividing total area into sub areas by lines bisecting every angle at the places of changes in geometry has been applied to all the models for optimal design of composite channel cross section. This method is shown in Figure 4.2.

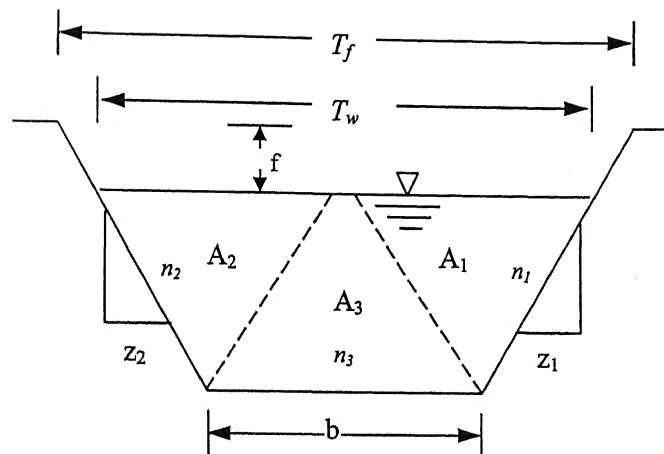


Figure 4.2: Trapezoidal channel section with bisecting lines of angle

The results of this methodology by Genetic Algorithm are presented in Tables 5.7 to 5.9 for Scenario-I, Scenario-II and Scenario-III respectively and the results by SQP (MATLAB 7.01) are given in Tables 5.10 to Tables 5.11 for Scenario-I, Scenario-II and Scenario-III respectively.

## 4.7 Proposed Optimization Model

The division of trapezoidal cross-section into sub-areas can be done in many ways as explained earlier. The simplest approach is to use vertical lines at the intersection of side slopes and/or change in roughness in the bed. However, when the velocities in the two adjoining areas are very different (as is the case looking at the results), then there exists a high velocity gradient at the assumed vertical imaginary boundary. The high velocity gradients at the imaginary vertical boundaries call for their inclusion into the total wetted

perimeter. This is because the high velocity gradients at the imaginary vertical boundaries essentially lead to an existence of shear stresses at such boundaries. Therefore, we need to modify the segmental wetted perimeters in different segments to take care of the resistance to flow at such boundaries. This can be achieved by partially incorporating the imaginary vertical distances corresponding to such boundaries through the use of certain coefficients in the segmental wetted perimeters. This can be achieved in many ways. This study investigates two different approaches. The first approach is based on Posey's (1967) approach. In this approach vertical imaginary boundaries are included to calculate the wetted perimeter in the middle sub section but not in the other sub sections. The second approach includes the imaginary vertical boundaries into all segments of a trapezoidal channel cross-section for the purpose of the calculation of the wetted perimeter. The calculations of the wetted perimeters according to the two approaches are described below.

#### **4.7.1 Approach-I: Model Q1**

This approach is based on Posey (1967) considerations for compound sections for calculating wetted perimeter. As per Posey (1967) criterion, the imaginary vertical boundaries are considered for the middle portion only and neglected completely in calculations of wetted perimeters for shallower portions. The incorporation of the imaginary vertical boundaries into the wetted perimeter calculations in the middle segment of the trapezoidal cross-section as per Posey (1967) criterion results in the following expressions for the segmental and total wetted perimeters.

$$P_{w1} = \sqrt{(z_1^2 + 1)} (y) \quad (4.60)$$

$$P_{w2} = \sqrt{(z_2^2 + 1)} (y) \quad (4.61)$$

$$P_{w3} = b + (k_1 + k_2) y \quad (4.62)$$

$$P_w = \left[ \sqrt{(z_1^2 + 1)} + \sqrt{(z_2^2 + 1)} + (k_1 + k_2) \right] y + b \quad (4.63)$$

where  $P_{w1}$ ,  $P_{w2}$ ,  $P_{w3}$  are the wetted perimeters (m) in sub-sections 1, 2, 3, respectively;  $P_w$  is the total wetted perimeter (m);  $y$  is the flow depth (m);  $z_1$  and  $z_2$  are the side slopes for the right and left sides of the trapezoidal cross-section, respectively; and  $k_1$  and  $k_2$  are the constants that account for the shear stresses at the vertical imaginary boundaries.

#### 4.7.2 Approach-II: Model Q2

The Posey (1967) criterion is normally used for compound channel sections consisting of a deeper middle portion and shallower portions on either side of the deeper section. Since the flow depth in the shallower portion in compound sections is small, the consideration of imaginary vertical boundary in wetted perimeter calculations for deeper segments only would not cause significant errors. However, in a composite trapezoidal channel cross-section, the maximum flow depth in all the segments is same. Therefore, neglecting the imaginary vertical boundary in wetted perimeter calculations for the right and left segments may be a source of error. A better approach may be to incorporate the imaginary vertical boundary into the wetted perimeter calculations for all segments of a composite trapezoidal section. This has been the motivation for the development of Approach-II in

this study. The incorporation of the imaginary vertical boundaries into the wetted perimeter calculations in all segments of the composite trapezoidal cross-section results in the following expressions for the segmental and total wetted perimeters.

$$P_{w1} = \sqrt{(z_1^2 + 1)} (y) + k_1 y \quad (4.64)$$

$$P_{w2} = \sqrt{(z_2^2 + 1)} (y) + k_2 y \quad (4.65)$$

$$P_{w3} = b + (k_1 + k_2) y \quad (4.66)$$

$$P_w = \left[ \sqrt{(z_1^2 + 1)} + \sqrt{(z_2^2 + 1)} + 2(k_1 + k_2) \right] y + b \quad (4.67)$$

#### 4.7.3 Determination of $k_1$ and $k_2$

The two approaches described above involve two unknown coefficients  $k_1$  and  $k_2$  that need to be determined. The question then arises is what the constants  $k_1$  and  $k_2$  represent physically and how we can determine these. These constants should be functions of the velocity gradients since the velocity gradients at the boundaries are responsible for producing shear stresses and hence certain amount of resistance to flow in the vicinity of such imaginary boundaries. This can be achieved by considering the constants to be directly proportional to the differences in the velocity between right and middle segments, and left and middle segments, respectively. In other words,  $k_1$  can be assumed to be a function of  $(V_3 - V_l)$  and  $k_2$  can be assumed to be a function of  $(V_3 - V_2)$ . Please note that we should take absolute magnitude of the differences in the velocities in adjoining segments. Further, the maximum value of the constants,  $k_1$  and  $k_2$ , will be 1.0 corresponding to the

case when  $z_1 = z_2 = 0.0$  i.e. when the trapezoidal cross section reduces to a rectangular cross-section. Further, as the value of  $z_1$  and/or  $z_2$  increases, the value of  $k_1$  and  $k_2$  is expected to decrease. This is because the effects of the rough side slopes on the flow field will tend to decrease as the side slopes go away from the imaginary vertical divides. The values of  $k_1$  and  $k_2$  would attain their minimum values (say  $k_{1\min}$  and  $k_{2\min}$ ) when the side slopes become horizontal or the trapezoidal channel becomes rectangular without vertical boundaries. At such point of time the differences in velocities in the adjoining segments would dictate the values of these coefficients. Based on these concepts, the constants  $k_1$  and  $k_2$  can be expressed by the following equations:

$$k_1 = \left| \frac{V_3 - V_1}{\text{Max}(V_3, V_1)} \right| \quad (4.68)$$

$$k_2 = \left| \frac{V_3 - V_2}{\text{Max}(V_3, V_2)} \right| \quad (4.69)$$

Where  $V_1$  and  $V_2$  are the segmental velocities in the right and left segments corresponding to the side slopes of  $z_1$  and  $z_2$ , respectively, and  $V_3$  is the segmental velocity in the middle segment of the composite trapezoidal channel. It is to be noted that  $V_1$ ,  $V_2$ , and  $V_3$  will be varying depending upon the side slopes and roughness coefficients in different sub areas of the composite channel carrying given discharge. The constants  $k_1$  and  $k_2$  would vary between  $k_{1\min}$  and 1.0 and  $k_{2\min}$  and 1.0, respectively. The higher limiting condition ( $k_1 = k_2 = 1.0$ ) occurs when  $z_1 = z_2 = 0.0$ , and  $V_1 = V_2 = 0.0$ ; and the lower limiting condition ( $k_1 = k_{1\min}$  and  $k_2 = k_{2\min}$ ) occurs when  $z_1 = z_2 = \infty$  (i.e. sides become horizontal).

The formulation of the constants  $k_1$  and  $k_2$  based on the principle of velocity gradients is presented by equations (4.68) and (4.69) above. However, the determination of  $k_1$  and  $k_2$  is not trivial since the segmental velocities  $V_1$  and  $V_2$  on the right hand side of equations (4.68) and (4.69) will be functions of the wetted perimeters of various segments, which in turn depends upon the values of  $k_1$  and  $k_2$  themselves. Therefore, the determination of  $k_1$  and  $k_2$  would involve an iterative procedure. The determination of  $k_1$  and  $k_2$  through an iterative procedure is summarized in the following steps:

1. Determine the segmental velocities  $V_1$ ,  $V_2$ , and  $V_3$  corresponding to the side slopes of  $z_1$  and  $z_2$  assuming the imaginary vertical boundaries as non-contributing to the wetted perimeter.
2. Estimate the initial values of  $k_1$  and  $k_2$  from equations (4.68) and (4.69), respectively.
3. Using the estimated values of  $k_1$  and  $k_2$ , compute the revised wetted perimeters  $P_{w1}$ ,  $P_{w2}$ , and  $P_{w3}$  using equations (4.60) through (4.62) for Approach-I and equations (4.64) through (4.66) for Approach-II.
4. Calculate the segmental areas ( $A_i$ 's) and determine the hydraulic radii ( $R_i = A_i / P_{wi}$ ) corresponding to the wetted perimeters computed in the previous step.
5. Knowing the roughness coefficients ( $n_1$ ,  $n_2$ ,  $n_3$ ) and the revised hydraulic radii ( $R_i$ 's) calculated in the previous step, calculate the revised segmental velocities ( $V_1$ ,  $V_2$ , and  $V_3$ ) using Manning's equation.
6. Repeat steps 2 through 5 until convergence is achieved in terms of the segmental velocities or total  $Q$ .

Once the convergence is achieved, the final values of the constants, velocities, and wetted perimeters can be employed in the solution procedure through GA to determine the optimal dimensions of the composite channel as done earlier. The results from the optimization problem from model Q1 and model Q2 are presented in Table 5.13.



# CHAPTER 5

## Results and Discussion

### 5.1 General

Depending upon the method and parameters required to calculate the equivalent roughness, various models can be divided into three groups. The first group involves models A to D which requires the relative area of the subsections and hence depends on the method of subsection but not on the relative length of the wetted perimeter. These models do not represent physically reasonable situation especially if equivalent roughness ( $n_c$ ) is viewed from momentum perspective. The optimal design costs of composite channel in these models depend mainly upon the method of subsections. The second group involves models from E to H. They depend only upon the relative length of wetted perimeter and do not count the area of subsections. Hence, all models of this group have no effect on optimal design cost by method of subsection. Third group involves models from I to Q which depends on subsection areas and also on relative length of wetted perimeters. Hence, these models are expected to be more sensitive than others.

## 5.2 Results from Method-1 Scenario I

Results from Method-1 Scenario-I that is division of total area into sub areas by vertical lines are given in Table 5.1. These models have been solved with GA. Results show that all the models of the first group are having approximately same cost varying from minimum cost 21.160 for Model C to maximum cost 21.241 for Model D. These costs are having much difference with the cost of models of another group. Hydraulic properties of all the models of this group are also similar. All the models of this group are having approximately same cross sectional area also varying from 28.925 m<sup>2</sup> for Model A to 29.092 m<sup>2</sup> for Model D. The models of the second group (Model E to Model H) are showing more or less same pattern. They have much difference with the models of another group but have quite similar values in the group ranging from minimum cost 22.702 for Model F to 23.026 for Model G. Cross sectional area of models of this group are also similar with minimum cross sectional area 31.454 for Model F to maximum area 31.893 for Model G. Third group is more sensitive on the changes in the conditions as within this group any change in the ratio of sub area to total area relative length of wetted perimeter causes a lot of change in the search space hence the optimal design cost of the composite channel and seems to be more logical on momentum perspective. Hence, we can say this group is more diversified group giving optimal cost of composite channel ranging from 14.379 for Model Q to 24.715 for Model I. These values are the best and the worst designing values of cost in all the groups.

To validate these results, these models have been also solved with the classical optimization technique, sequential quadratic programming (SQP) of MATLAB 7.01 optimization tool box. The results are given in Table 5.4. This method is also giving similar values as in GA. Solutions by this method show that the cost values for the first group are ranging from 21.136 to 21.158 that is quite similar to the results with GA. For the second group these results range from minimum 22.685 for Model G to a maximum value 23.012 for Model H. Similar to the results of GA, this method is also giving the minimum cost of construction 14.341 for Model Q.

Hence, it is clear from the results in Table 5.1 that Model Q is giving the minimum value of cost. This value is 30.96% less than the average value of all these models. Hence, we can say that trapezoidal channel designed by Model Q is the most efficient than the other models. Channel parameters (width, depth, and slopes of the two sides) and hydraulic parameters (area, wetted perimeter) are also given in Tables 5.1 for results with GA and in Table 5.4 for results with SQP. Results show that Model Q has minimum cross sectional area  $18.723 \text{ m}^2$  which is 34.27% less than the average value of all these models.

### 5.3 Results from Method-1 Scenario II

All the models have been solved for two other scenarios. Scenario II represent the designing of composite channel for all the models when there is present limited right of way in the field and we have to restrict the top width of channels. In this study, a constant value of top width equal to 7.5 m is taken as the upper limit to design the channel cross

section. For this purpose, an extra constraint represented by equation (4.55) was added to the optimization problem that restricts the top width to the maximum permissible top width. Results from Method -1 Scenario II that is models solved with method of division of total area into sub areas by vertical lines and having an additional constraint, equation (4.55), solution by GA are given in Table 5.2. Results show that there is no change in the optimal design of the models of the first group as they all are having top width less than maximum permissible. They are having the same results as in Scenario I. Models of the second group are having higher values of the cost than in Scenario I but they have approximately similar values for all models ranging from minimum value 22.726 for Model F to maximum value 23.066 for Model G. In this scenario also, Model Q has lowest value of cost in all the models that is 14.379 as in Scenario I. In this scenario, the average cost of all the models is higher than Scenario I and Model Q is 31.58% lesser than the average of all the models. In this scenario also, Model Q has least area of cross section ( $18.723 \text{ m}^2$ ) that is 34.84% lesser than the average value of all these models. The results from classical method are given in Table 5.4. These results are approximately similar to the results of GA.

## 5.4 Results from Method-1 Scenario III

All the models have been solved for another scenario that is restricted side slope. In this scenario optimization problem is subjected to some additional constraints, equations (4.56) to (4.59). Results solving all the models with GA are given in Table 5.3. In this method also, the models of first group are showing similar pattern as in the previous two scenarios.

These models have approximately same values of design cost having minimum value 23.452 for Model C and maximum value 23.787 for Model B. In this scenario all the models of this group having higher cost than other two scenarios. Cross sectional area for these models are also approximately same. Models of second group are also showing similar pattern. They have design cost ranging from 24.330 for Model F to 24.615 for Model G. In this scenario, Model Q and Model L have least cost in all models that is 20.931. This value is 11.20% lesser than the average of all the models. In this scenario, almost all of the models have higher value of design cost than the two scenarios excluding Model I that has higher cost in Scenario 2. The results from all the models for Scenario III solved with classical method SQP are given in Table 5.6. These results are also giving approximately same values as calculated with the help of GA.

## 5.5 Results from Method-2 Scenario I

Results from Method-2 Scenario I that is division of total area into sub areas by bisecting lines at every change in geometry or roughness are given in Table 5.7. These are the results with GA. In this method also, the models from the first group having approximately same value with minimum cost of 22.674 for Model C up to the maximum value 22.976 for Model B. Similarly, models of second group are also showing similar pattern having minimum value of cost 22.702 for Model F and maximum cost of 23.026 for Model G. In this method also, Model Q is having the lowest cost that is 22.453 which is 2.66% lower than the average value of all the models. While comparing these results with that of Method-1 Scenario I of Table 5.1 we are getting some very interesting results.

Models of first group are giving slightly higher cost that means this method of division of total area into sub areas is giving over designed values for the models of this group. There is no change in the cost of models of second group. It is due to the fact that they are not affected by the method of sub area division as explained earlier. For the models of the third group, this method is giving very high value of cost for all the models. One another conclusion that can be made while comparing these results with that of Method-1 is that excluding Model I (which is giving very high value in both the results) all the models are giving approximately similar values in Method-2 in comparison to Method-1 as standard deviation for cost of all the models for this method is 1.056 as that for all the models in Method-1 was 2.964. Hence, we can conclude that the results for all models in this method are near to the average value as compared to Method-1. For the Model Q which is performing in the best manner in the two methods, in the second method cross sectional area is  $30.963 \text{ m}^2$  which is 1.654 times higher than that in Method-1. the results all the models for Method-2 Scenario I solved with classical method (SQP) are given in Table 5.10. While comparing with GA results, these results are giving approximately similar values.

## 5.6 Results from Method-2 Scenario II

Results for Method-2 Scenario II are given in Table 5.8. While comparing to Method-2 Scenario I, Table 5.7, all the models of the first, second and third groups are having slightly higher values of cost as compared to Scenario I. Model I has the highest value of cost 27.139 as Model Q has lowest one 22.885. When these results are compared with that

of Method-1 Scenario 2, Table 5.2 the following conclusions can be made. Models of group one have slightly higher values of cross sectional area and also the design cost. There is no change in the results of the second group. Members of the third group are giving very high cost. These results from all the models for Method-2 Scenario II solved with classical method are given in Table 5.11. These results are approximately similar to that of GA in all respects.

## 5.7 Results from Method-2 Scenario III

Results for all the models for Method-2 Scenario III are given in the Table 5.9. These are the results with GA. All the models are having approximately similar cost. Least cost Model is Model Q with a cost of 23.524 and highest cost Model is Model I with a cost of 29.076. While comparing within the results of the Table 5.9, Model Q has 4.22% lesser than the average value of all the models. While comparing with the other two scenarios of Method-2, all the models are having higher value than other two scenarios. When these results are compared with the results of Method-1 Scenario III, Table 5.3 the following interesting result may be noted. Costs of models of the first groups in this method are slightly higher than that of Method-1. Cost of second group of models remains unchanged and cost becomes very high in the case of models of third group. Method-2 scenario 3 for all the models have been solved with the classical method (SQP from MATLAB 7.01 optimization toolbox) and the results are presented in Table 5.12. While comparing with the results of GA, all the models are having approximately similar values as in GA and hydraulic parameters are also similar.

## 5.8 Results and discussion for Model Q1 and Model Q2

From the previous results, it is clear that the Model Q is the best Model for designing least cost composite channel cross section. Model Q represent the flow conditions in the channel more accurately due to following reasons:

- (a) Allowing spatial variation in segmental roughness coefficient as using a single equivalent roughness coefficient.
- (b) Allowing spatial variation in velocity across the composite channel cross section
- (c) Allowing for incorporating perimeter along imaginary line.

In the above Methods A to P, for designing optimal cross section of composite channel, it was considered that the dividing line is a zero shear line and there is no resistance along this line. But Model Q is based upon the spatial variation of velocity across the cross section and from the observations of velocity in the different subsections, it is clear that there is large differences in velocities in two sub areas divided by vertical lines. Hence, there will be a large velocity gradient along the vertical line that is liable to modify the resistance to the flow. To calculate the effect of the vertical imaginary line on the optimal design cost of the composite trapezoidal channel, two approaches have been developed in this study. Cost of all three scenarios with approach 1 (Model Q1) and approach 2 (Model Q2) are given in Table 5.13. In this case, it is clear that as we are considering the resistance along the vertical imaginary line of the Model Q, resistance to flow will be increased and hence a larger size of channel will be required to fulfill the need of the



required design discharge. In approach 2, since, the resistance to flow is more, slightly higher values of designed parameters are expected that increases the cost of construction. The values of channel parameters for the two approaches are presented in Table 5.13. Results show that Model Q1 has cost value 21.384 for Scenario I and Scenario II which is slightly lower than that of Model Q2 which has cost value 21.443 for Scenario I and Scenario II. In case of Scenario III Model Q1 has cost value 24.010 that is slightly lower than cost value of Model Q2 for Scenario III which is 24.309. Since in the total cost between the two approaches is not significant, and the second approach is more accurate, it is recommended for optimal design of composite channel. About the channel parameters it is clear from the results Table 5.13 that Model Q2 has larger cross sectional area in all three scenarios than that of Model Q1 but it has lower value of wetted perimeter than Model Q1. These may be due to the fact that we have considered larger resistances to flow at the vertical imaginary lines in Model Q2 than in Model Q1. Hence, it requires larger area to carry designed discharge.

Table 5.1 Results from various models for vertical line division method, scenario I solved with GA

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	5.569	4.235	0.025	0.203	0.280	2.276	26.369	28.925	4.736	4.832	5.569	15.137	6.65	21.185
B	5.746	4.159	0.044	0.156	0.478	1.689	26.771	28.937	4.664	4.715	5.746	15.125	6.68	21.201
C	5.663	4.100	0.127	0.146	1.344	1.545	26.050	28.938	4.637	4.649	5.663	14.949	6.92	21.160
D	5.801	3.972	0.087	0.228	0.870	2.280	25.942	29.092	4.489	4.587	5.801	14.877	7.21	21.241
E	5.395	4.235	0.237	0.329	11.521	11.801	8.568	31.890	4.866	4.985	5.395	15.246	8.08	22.976
F	5.774	3.964	0.339	0.231	10.521	10.226	10.708	31.454	4.714	4.582	5.774	15.069	8.32	22.702
G	6.146	3.884	0.245	0.27	9.894	9.954	12.045	31.893	4.514	4.541	6.146	15.201	8.40	23.026
H	6.325	3.99	0.14	0.183	10.178	10.247	11.229	31.655	4.534	4.565	6.325	15.423	7.78	22.939
I	6.304	3.757	0.394	0.458	3.570	4.150	26.836	34.556	4.576	4.682	6.304	15.562	9.93	24.715
J	6.980	3.669	0.158	0.003	1.373	0.026	29.100	30.499	4.221	4.169	6.980	15.370	7.65	22.280
K	6.179	3.637	0.021	0.120	0.180	1.027	25.563	26.769	4.138	4.167	6.179	14.484	6.76	19.792
L	4.124	3.738	0.034	0.142	0.306	1.275	17.478	19.059	4.240	4.281	4.124	12.645	4.87	14.592
M	4.790	4.269	0.178	0.342	2.024	3.889	22.844	28.757	4.844	5.040	4.790	14.674	7.27	20.918
N	6.029	3.593	0.565	0.533	4.733	4.465	24.677	33.874	4.701	4.638	6.029	15.368	10.52	24.241
O	5.457	3.811	0.035	0.033	0.321	0.307	23.525	24.152	4.314	4.313	5.457	14.084	5.75	18.072
P	5.911	3.631	0.040	0.036	0.341	0.307	24.418	25.067	4.134	4.134	5.911	14.179	6.22	18.679
Q	3.737	3.953	0.046	0.164	0.456	1.626	16.641	18.723	4.458	4.512	3.737	12.707	4.67	14.379

Table 5.2 Results from various models for vertical line division method, scenario II solved with GA

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	5.569	4.235	0.025	0.203	0.280	2.276	26.369	28.925	4.736	4.832	5.569	15.137	6.65	21.185
B	5.746	4.159	0.044	0.156	0.478	1.689	26.771	28.937	4.664	4.715	5.746	15.125	6.68	21.201
C	5.663	4.100	0.127	0.146	1.344	1.545	26.050	28.938	4.637	4.649	5.663	14.949	6.92	21.160
D	5.801	3.972	0.087	0.228	0.870	2.280	25.942	29.092	4.489	4.587	5.801	14.877	7.21	21.241
E	5.749	4.306	0.169	0.195	11.713	11.766	8.355	31.833	4.874	4.897	5.749	15.520	7.50	23.003
F	5.822	4.211	0.207	0.149	11.332	11.219	8.827	31.378	4.811	4.763	5.822	15.396	7.50	22.726
G	5.798	4.309	0.199	0.154	11.790	11.700	8.475	31.964	4.903	4.866	5.798	15.567	7.50	23.066
H	6.018	4.189	0.137	0.178	11.096	11.166	9.419	31.681	4.733	4.763	6.018	15.514	7.50	22.950
I	3.798	5.795	0.305	0.279	6.043	5.528	23.908	35.480	6.581	6.535	3.798	16.915	7.47	25.382
J	6.866	3.728	0.138	0.012	1.233	0.107	29.030	30.371	4.268	4.228	6.866	15.362	7.50	22.301
K	6.179	3.637	0.021	0.120	0.180	1.027	25.563	26.769	4.138	4.167	6.179	14.484	6.76	19.792
L	4.124	3.738	0.034	0.142	0.306	1.275	17.478	19.059	4.240	4.281	4.124	12.645	4.87	14.592
M	4.790	4.269	0.178	0.342	2.024	3.889	22.844	28.757	4.844	5.040	4.790	14.674	7.27	20.918
N	5.376	5.388	0.055	0.267	0.953	4.628	31.654	37.236	5.897	6.094	5.376	17.367	7.27	26.670
O	5.457	3.811	0.035	0.033	0.321	0.307	23.525	24.152	4.314	4.313	5.457	14.084	5.75	18.072
P	5.911	3.631	0.040	0.036	0.341	0.307	24.418	25.067	4.134	4.134	5.911	14.179	6.22	18.679
Q	3.737	3.953	0.046	0.164	0.456	1.626	16.641	18.723	4.458	4.512	3.737	12.707	4.67	14.379

Table 5.3 Results from various models for vertical line division method, scenario III solved with GA

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	4.028	3.582	1.000	1.000	8.331	8.331	16.442	33.105	5.773	5.773	4.028	15.574	12.19	23.673
B	4.112	3.567	1.000	1.000	8.270	8.270	16.724	33.264	5.752	5.752	4.112	15.615	12.25	23.787
C	4.205	3.494	1.000	1.000	7.976	7.976	16.795	32.747	5.648	5.648	4.205	15.502	12.19	23.452
D	4.083	3.567	1.000	1.000	8.270	8.270	16.606	33.146	5.752	5.752	4.083	15.586	12.22	23.703
E	3.777	3.782	1.000	1.000	12.965	12.965	8.578	34.509	6.056	6.056	3.777	15.888	12.34	24.567
F	3.71	3.776	1.000	1.000	12.929	12.929	8.290	34.148	6.047	6.047	3.710	15.804	12.26	24.330
G	3.708	3.813	1.000	1.000	13.154	13.154	8.287	34.595	6.100	6.100	3.708	15.907	12.33	24.615
H	3.817	3.763	1.000	1.000	12.850	12.850	8.744	34.445	6.029	6.029	3.817	15.875	12.34	24.524
I	2.836	4.208	1.000	1.000	11.083	11.083	13.352	35.517	6.658	6.658	2.836	16.152	12.25	25.161
J	9.831	6.820	1.000	1.000	26.791	26.791	71.963	125.545	10.352	10.352	9.831	30.535	24.47	82.937
K	4.085	3.969	1.000	1.000	9.986	9.986	18.256	38.228	6.320	6.320	4.085	16.725	13.02	22.939
L	3.609	3.372	1.000	1.000	7.496	7.496	13.974	28.966	5.476	5.476	3.609	14.561	11.35	20.931
M	3.823	3.557	1.000	1.000	8.230	8.230	15.510	31.969	5.737	5.737	3.823	15.298	11.94	22.916
N	3.360	3.984	1.000	1.000	10.053	10.053	15.066	35.172	6.341	6.341	3.360	16.043	12.33	24.972
O	4.757	3.338	1.000	1.000	7.365	7.365	18.257	32.988	5.428	5.428	4.757	15.613	12.43	23.667
P	4.131	3.567	1.000	1.000	8.270	8.270	16.801	33.341	5.752	5.752	4.131	15.634	12.27	23.841
Q	3.542	3.396	1.000	1.000	7.589	7.589	13.800	28.978	5.510	5.510	3.542	14.562	11.33	20.931

**Table 5.5 Results from various models for vertical line division method, scenario II solved with classical method**

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	5.829	4.279	0.000	0.079	0.000	0.902	27.857	28.759	4.779	4.794	5.829	15.402	6.21	21.152
B	5.877	4.287	0.000	0.054	0.000	0.619	28.133	28.752	4.787	4.794	5.877	15.458	6.14	21.158
C	5.712	4.266	0.017	0.121	0.193	1.374	27.223	28.791	4.767	4.801	5.712	15.279	6.37	21.136
D	5.851	4.281	0.000	0.067	0.000	0.766	27.974	28.739	4.781	4.792	5.851	15.424	6.17	21.155
E	5.854	4.261	0.167	0.179	1.893	2.029	27.871	31.792	4.827	4.837	5.854	15.518	7.50	23.002
F	5.862	4.194	0.171	0.178	1.884	1.961	27.516	31.361	4.762	4.768	5.862	15.392	7.50	22.721
G	5.834	4.277	0.167	0.182	1.905	2.077	27.869	31.851	4.843	4.855	5.834	15.533	7.50	23.058
H	5.861	4.245	0.167	0.178	1.880	2.004	27.810	31.694	4.811	4.820	5.861	15.491	7.50	22.946
I	5.123	4.864	0.443	0.000	6.373	0.000	27.480	33.853	5.867	5.364	5.123	16.354	7.50	24.375
J	7.326	3.617	0.029	0.003	0.246	0.025	30.161	30.432	4.119	4.117	7.326	15.562	7.46	22.318
K	5.881	3.774	0.080	0.128	0.731	1.169	25.135	27.035	4.288	4.309	5.881	14.478	6.77	19.791
L	4.054	3.769	0.017	0.172	0.155	1.567	17.307	19.029	4.270	4.332	4.054	12.655	4.86	14.570
M	4.909	4.289	0.212	0.239	2.431	2.741	23.509	28.681	4.895	4.924	4.909	14.728	7.07	20.892
N	2.364	6.972	0.383	0.305	10.692	8.514	17.664	36.870	8.001	7.812	2.364	18.177	7.50	26.372
O	5.261	3.935	0.026	0.049	0.256	0.482	23.333	24.070	4.436	4.440	5.261	14.138	5.59	18.022
P	5.105	4.167	0.029	0.083	0.316	0.904	23.825	25.045	4.669	4.683	5.105	14.457	5.63	18.675
Q	3.891	3.862	0.060	0.171	0.571	1.627	16.973	19.170	4.370	4.425	3.891	12.686	4.90	14.341

**Table 5.6 Results from various models for vertical line division method, scenario III solved with classical method**

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	4.166	3.535	1.000	1.000	8.141	8.141	16.810	33.091	5.706	5.706	4.166	15.579	12.24	23.675
B	4.184	3.544	1.000	1.000	8.177	8.177	16.920	33.274	5.719	5.719	4.184	15.622	12.27	23.791
C	4.092	3.533	1.000	1.000	8.133	8.133	16.503	32.768	5.704	5.704	4.092	15.499	12.16	23.454
D	4.177	3.539	1.000	1.000	8.157	8.157	16.871	33.184	5.712	5.712	4.177	15.601	12.26	23.733
E	3.776	3.783	1.000	1.000	9.172	9.172	16.173	34.517	6.057	6.057	3.776	15.890	12.34	24.571
F	3.966	3.686	1.000	1.000	8.761	8.761	16.602	34.124	5.920	5.920	3.966	15.806	12.34	24.326
G	3.729	3.806	1.000	1.000	9.271	9.271	16.057	34.599	6.090	6.090	3.729	15.908	12.34	24.615
H	3.820	3.762	1.000	1.000	9.082	9.082	16.281	34.445	6.027	6.027	3.820	15.875	12.34	24.524
I	4.027	3.814	1.000	1.000	9.305	9.305	17.372	35.983	6.101	6.101	4.027	16.229	12.66	25.545
J	9.828	6.821	1.000	1.000	26.799	26.799	71.951	125.548	10.353	10.353	6.821	27.528	24.470	82.921
K	4.028	3.490	1.000	1.000	7.960	7.960	16.072	31.992	5.643	5.643	4.028	15.313	12.01	22.940
L	3.610	3.373	1.000	1.000	7.500	7.500	13.982	28.982	5.477	5.477	3.610	14.564	11.36	20.932
M	3.784	3.571	1.000	1.000	8.287	8.287	15.405	31.978	5.757	5.757	3.784	15.299	11.93	22.916
N	1.334	4.877	1.000	1.000	14.456	14.456	7.173	36.085	7.604	7.604	1.334	16.542	12.09	25.471
O	0.352	5.292	1.000	1.000	16.774	16.774	2.039	35.586	8.191	8.191	0.352	16.734	11.94	25.143
P	4.126	3.569	1.000	1.000	8.278	8.278	16.789	33.345	5.754	5.754	4.126	15.635	12.26	23.841
Q	3.609	3.373	1.000	1.000	7.500	7.500	13.978	28.978	5.477	5.477	3.609	14.563	11.36	20.931

**Table 5.7 Results from various models for bisecting line division method, scenario I solved with GA**

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	6.677	3.512	0.272	0.329	8.340	8.472	14.812	31.625	4.158	4.224	6.677	15.058	9.09	22.874
B	6.339	3.663	0.285	0.343	9.010	9.161	13.660	31.831	4.329	4.401	6.339	15.069	8.95	22.976
C	6.892	3.561	0.193	0.201	8.398	8.411	14.428	31.237	4.136	4.142	6.892	15.170	8.49	22.674
D	6.602	3.581	0.229	0.343	8.543	8.804	14.360	31.706	4.187	4.314	6.602	15.103	8.94	22.925
E	5.395	4.235	0.237	0.329	11.521	11.801	8.568	31.890	4.866	4.985	5.395	15.246	8.08	22.976
F	5.774	3.964	0.339	0.231	10.521	10.226	10.708	31.454	4.714	4.582	5.774	15.069	8.32	22.702
G	6.146	3.884	0.245	0.27	9.894	9.954	12.045	31.893	4.514	4.541	6.146	15.201	8.40	23.026
H	6.325	3.99	0.14	0.183	10.178	10.247	11.229	31.655	4.534	4.565	6.325	15.423	7.78	22.939
I	6.703	4.628	0.086	0.184	13.197	13.369	11.357	37.923	5.147	5.214	6.703	17.064	8.09	27.097
J	5.982	3.913	0.256	0.312	10.051	10.200	11.678	31.929	4.555	4.623	5.982	15.160	8.49	23.020
K	6.228	3.721	0.183	0.385	9.056	9.546	12.746	31.348	4.291	4.523	6.228	15.042	8.63	22.667
L	6.962	3.211	0.446	0.301	7.540	7.191	16.249	30.980	4.063	3.875	6.962	14.901	9.73	22.467
M	6.686	3.487	0.268	0.294	8.229	8.284	14.611	31.124	4.128	4.156	6.686	14.969	8.93	22.550
N	6.041	3.892	0.228	0.314	9.892	10.109	11.758	31.760	4.505	4.603	6.041	15.149	8.42	22.927
O	6.621	3.629	0.214	0.296	8.717	8.890	14.078	31.686	4.222	4.306	6.621	15.150	8.73	22.919
P	5.827	4.013	0.212	0.327	10.410	10.714	10.662	31.786	4.613	4.748	5.827	15.188	8.26	22.929
Q	7.038	3.215	0.387	0.311	7.399	7.227	16.337	30.963	3.983	3.891	7.038	14.912	9.63	22.453

Table 5.8 Results from various models for bisecting line division method, scenario II solved with GA

Model	b	y	$z_1$	$z_2$	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	6.703	3.974	0.05	0.129	10.021	10.091	11.669	31.781	4.480	4.511	6.703	15.694	7.50	23.094
B	6.535	4.056	0.083	0.128	10.414	10.463	11.086	31.963	4.572	4.593	6.535	15.700	7.50	23.209
C	6.354	4.052	0.108	0.143	10.421	10.466	10.638	31.524	4.578	4.598	6.354	15.531	7.50	22.865
D	6.545	4.047	0.066	0.144	10.360	10.444	11.127	31.931	4.557	4.594	6.545	15.696	7.50	23.156
E	5.749	4.306	0.169	0.195	11.713	11.766	8.355	31.833	4.874	4.897	5.749	15.520	7.50	23.003
F	5.822	4.211	0.207	0.149	11.332	11.219	8.827	31.378	4.811	4.763	5.822	15.396	7.50	22.726
G	5.798	4.309	0.199	0.154	11.790	11.700	8.475	31.964	4.903	4.866	5.798	15.567	7.50	23.066
H	6.018	4.189	0.137	0.178	11.096	11.166	9.419	31.681	4.733	4.763	6.018	15.514	7.50	22.950
I	6.715	4.828	0.068	0.079	14.227	14.238	9.399	37.864	5.340	5.345	6.715	17.400	7.50	27.139
J	6.541	4.046	0.086	0.124	10.371	10.412	11.122	31.905	4.563	4.581	6.541	15.685	7.50	23.168
K	6.469	4.005	0.108	0.121	10.207	10.222	11.039	31.467	4.531	4.538	6.469	15.538	7.50	22.863
L	6.692	3.932	0.098	0.085	9.868	9.857	11.731	31.456	4.453	4.448	6.692	15.593	7.50	22.884
M	6.859	3.878	0.096	0.051	9.628	9.596	12.214	31.437	4.398	4.384	6.859	15.641	7.50	22.894
N	6.368	4.062	0.097	0.152	10.455	10.525	10.662	31.642	4.583	4.614	6.368	15.566	7.50	22.960
O	5.927	4.235	0.123	0.209	11.295	11.452	9.039	31.786	4.771	4.837	5.927	15.535	7.50	23.024
P	5.786	4.284	0.163	0.195	11.594	11.659	8.524	31.777	4.847	4.874	5.786	15.507	7.50	22.993
Q	6.641	3.949	0.101	0.092	9.947	9.939	11.570	31.456	4.472	4.468	6.641	15.580	7.50	22.885



**Table 5.9 Results from various models for bisectioning line division method, scenario III solved with GA**

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	4.768	3.413	1.000	1.000	10.827	10.827	12.315	33.969	5.534	5.534	4.768	15.836	12.59	24.310
B	4.786	3.422	1.000	1.000	10.877	10.877	12.399	34.153	5.547	5.547	4.786	15.879	12.63	24.422
C	4.828	3.668	1.000	1.000	12.284	12.284	12.927	37.495	5.894	5.894	4.828	16.617	13.16	24.083
D	4.818	3.404	1.000	1.000	10.777	10.777	12.496	34.051	5.521	5.521	4.818	15.860	12.63	24.364
E	3.777	3.782	1.000	1.000	12.965	12.965	8.578	34.509	6.056	6.056	3.777	15.888	12.34	24.567
F	3.71	3.776	1.000	1.000	12.929	12.929	8.290	34.148	6.047	6.047	3.710	15.804	12.26	24.330
G	3.708	3.813	1.000	1.000	13.154	13.154	8.287	34.595	6.100	6.100	3.708	15.907	12.33	24.615
H	3.817	3.763	1.000	1.000	12.850	12.850	8.744	34.445	6.029	6.029	3.817	15.875	12.34	24.524
I	3.508	4.413	1.000	1.000	17.068	17.068	7.237	41.372	6.948	6.948	3.508	17.404	13.33	29.076
J	4.901	3.394	1.000	1.000	10.722	10.722	12.804	34.248	5.507	5.507	4.901	15.915	12.69	24.497
K	7.203	2.728	1.000	1.000	7.368	7.368	18.935	33.671	4.565	4.565	7.203	16.333	13.66	24.427
L	5.403	3.121	1.000	1.000	9.271	9.271	14.133	32.676	5.121	5.121	5.403	15.645	12.65	23.530
M	5.32	3.188	1.000	1.000	9.618	9.618	13.986	33.222	5.216	5.216	5.320	15.751	12.70	23.877
N	3.487	3.889	1.000	1.000	13.621	13.621	7.325	34.568	6.207	6.207	3.487	15.901	12.27	24.541
O	4.887	3.387	1.000	1.000	10.684	10.684	12.738	34.105	5.497	5.497	4.887	15.881	12.66	24.404
P	4.471	3.528	1.000	1.000	11.473	11.473	11.289	34.234	5.696	5.696	4.471	15.864	12.53	24.445
Q	5.607	3.059	1.000	1.000	8.957	8.957	14.709	32.622	5.033	5.033	5.607	15.673	12.73	23.524

Table 5.10 Results for various models for bisecting line division method, scenario I solved with classical method

Model	b	y	$z_1$	$z_2$	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	6.449	3.64	0.281	0.296	8.901	8.937	13.80	31.64	4.300	4.317	6.449	15.07	8.837	22.860
B	6.454	3.656	0.28	0.299	8.968	9.014	13.84	31.82	4.315	4.337	6.454	15.11	8.860	22.973
C	6.398	3.627	0.284	0.293	8.853	8.874	13.59	31.31	4.290	4.301	6.398	14.99	8.779	22.639
D	6.453	3.647	0.28	0.297	8.930	8.970	13.82	31.72	4.306	4.326	6.453	15.08	8.845	22.916
E	5.826	4.052	0.247	0.265	10.671	10.718	10.43	31.82	4.688	4.709	5.826	15.22	8.156	22.958
F	5.84	4.005	0.246	0.256	10.450	10.474	10.47	31.40	4.639	4.650	5.84	15.13	8.101	22.684
G	5.814	4.065	0.248	0.267	10.735	10.785	10.38	31.90	4.703	4.724	5.814	15.24	8.164	23.012
H	5.835	4.04	0.246	0.263	10.613	10.656	10.46	31.73	4.675	4.694	5.835	15.20	8.145	22.904
I	6.843	4.552	0.122	0.139	12.855	12.884	12.16	37.90	5.089	5.100	6.843	17.03	8.161	27.088
J	6.312	3.748	0.271	0.290	9.348	9.395	13.13	31.87	4.401	4.423	6.312	15.13	8.695	23.003
K	6.495	3.556	0.295	0.303	8.576	8.595	14.09	31.26	4.228	4.238	6.495	14.96	8.920	22.610
L	6.796	3.308	0.35	0.360	7.682	7.705	15.63	31.02	4.034	4.047	6.796	14.87	9.499	22.438
M	6.567	3.491	0.308	0.314	8.333	8.347	14.48	31.16	4.176	4.183	6.567	14.92	9.049	22.546
N	5.763	4.087	0.244	0.260	10.829	10.870	10.03	31.74	4.721	4.739	5.763	15.22	8.074	22.894
O	6.248	3.774	0.267	0.283	9.453	9.492	12.78	31.73	4.423	4.441	6.248	15.11	8.594	22.902
P	6.118	3.859	0.259	0.276	9.814	9.855	12.08	31.75	4.502	4.521	6.118	15.14	8.450	22.910
Q	6.796	3.308	0.349	0.354	7.679	7.691	15.60	30.97	4.033	4.039	6.796	14.87	9.473	22.437

Table 5.11 Results for various models for bisecting line division method, scenario II solved with classical method

Model	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	6.596	4.009	0.097	0.103	10.21	10.21	11.34	31.77	4.53	4.53	6.59	15.65	7.50	23.086
B	6.584	4.039	0.097	0.105	10.34	10.35	11.25	31.96	4.56	4.56	6.58	15.70	7.50	23.204
C	6.578	3.965	0.101	0.105	10.01	10.02	11.38	31.42	4.48	4.48	6.57	15.55	7.50	22.849
D	6.592	4.023	0.097	0.104	10.27	10.28	11.31	31.87	4.54	4.54	6.59	15.68	7.50	23.146
E	5.854	4.261	0.167	0.179	11.49	11.52	8.78	31.79	4.82	4.83	5.85	15.51	7.50	23.002
F	5.862	4.194	0.171	0.178	11.17	11.18	8.99	31.36	4.76	4.76	5.86	15.39	7.50	22.721
G	5.844	4.277	0.167	0.18	11.56	11.59	8.71	31.87	4.84	4.85	5.84	15.54	7.50	23.057
H	5.861	4.245	0.167	0.179	11.41	11.43	8.85	31.70	4.81	4.82	5.86	15.49	7.50	22.946
I	7.037	4.699	0.042	0.047	13.52	13.52	10.73	37.78	5.20	5.20	7.03	17.44	7.50	27.128
J	6.337	4.117	0.121	0.131	10.73	10.74	10.45	31.94	4.65	4.65	6.33	15.64	7.50	23.161
K	6.626	3.95	0.096	0.1	9.94	9.95	11.52	31.42	4.47	4.47	6.62	15.56	7.50	22.859
L	6.74	3.917	0.084	0.088	9.78	9.79	11.86	31.44	4.43	4.43	6.74	15.60	7.50	22.884
M	6.872	3.874	0.071	0.073	9.59	9.59	12.25	31.43	4.38	4.38	6.87	15.64	7.50	22.892
N	5.896	4.229	0.164	0.175	11.33	11.35	8.99	31.67	4.79	4.80	5.89	15.48	7.50	22.932
O	6.138	4.148	0.142	0.151	10.91	10.92	9.85	31.69	4.69	4.70	6.13	15.53	7.50	23.016
P	5.983	4.209	0.156	0.166	11.22	11.23	9.28	31.74	4.76	4.77	5.98	15.52	7.50	22.910
Q	6.740	3.917	0.084	0.088	9.789	9.793	11.866	31.448	4.433	4.434	6.740	15.607	7.50	22.884

**Table 5.12 Results for various models for bisecting line division method, scenario III solved with classical method**

Model	b	y	$z_1$	$z_2$	A1	A2	A3	A	P1	P2	P3	P	T	Cost
A	4.786	3.408	1.000	1.000	10.79	10.79	12.37	33.97	5.526	5.526	4.786	15.839	12.60	24.311
B	4.746	3.435	1.000	1.000	10.95	10.95	12.26	34.15	5.564	5.564	4.746	15.875	12.61	24.423
C	4.806	3.374	1.000	1.000	10.61	10.61	12.40	33.62	5.478	5.478	4.806	15.763	12.55	24.084
D	4.769	3.421	1.000	1.000	10.87	10.87	12.33	34.07	5.545	5.545	4.769	15.859	12.61	24.367
E	3.776	3.783	1.000	1.000	12.97	12.97	8.57	34.51	6.057	6.057	3.776	15.890	12.34	24.571
F	3.965	3.686	1.000	1.000	12.39	12.39	9.33	34.12	5.919	5.919	3.965	15.801	12.33	24.326
G	3.729	3.806	1.000	1.000	13.11	13.11	8.37	34.59	6.089	6.089	3.729	15.908	12.34	24.615
H	3.82	3.762	1.000	1.000	12.84	12.84	8.75	34.44	6.027	6.027	3.82	15.874	12.34	24.524
I	3.776	4.322	1.000	1.000	16.44	16.44	8.57	41.45	6.819	6.819	3.776	17.414	13.42	29.072
J	4.585	3.496	1.000	1.000	11.29	11.29	11.70	34.28	5.651	5.651	4.585	15.887	12.57	24.489
K	5.03	3.291	1.000	1.000	10.16	10.16	13.11	33.44	5.361	5.361	5.03	15.752	12.61	23.984
L	5.614	3.058	1.000	1.000	8.951	8.95	14.73	32.63	5.031	5.031	5.614	15.677	12.73	23.523
M	5.068	3.265	1.000	1.000	10.02	10.02	13.20	33.25	5.324	5.324	5.068	15.717	12.59	23.869
N	3.467	3.892	1.000	1.000	13.63	13.63	7.23	34.51	6.211	6.211	3.467	15.889	12.25	24.541
O	4.648	3.464	1.000	1.000	11.11	11.11	11.91	34.13	5.605	5.605	4.648	15.859	12.57	24.399
P	4.463	3.531	1.000	1.000	11.49	11.49	11.25	34.23	5.701	5.701	4.463	15.864	12.52	24.445
Q	5.614	3.058	1.000	1.000	8.952	8.952	14.731	32.634	5.032	5.032	5.614	15.678	12.73	23.523

**Table 5.13 Results for Model Q1 and Q2**

Model	Scenario	b	y	z <sub>1</sub>	z <sub>2</sub>	A1	A2	A3	A	P1	P2	P3	P	T	Cost
Q1	I	5.551	4.537	0.045	0.043	0.571	0.545	27.960	29.077	5.042	5.042	5.551	15.635	5.99	21.384
Q1	II	5.551	4.537	0.045	0.043	0.571	0.545	27.960	29.077	5.042	5.042	5.551	15.635	5.99	21.384
Q1	III	1.190	4.753	1.000	1.000	13.797	13.797	6.251	33.845	7.429	7.429	1.190	16.048	11.70	24.010
Q2	I	5.184	4.731	0.019	0.266	0.260	3.639	27.118	31.017	5.232	5.413	4.731	15.376	6.675	21.443
Q2	II	5.184	4.731	0.019	0.266	0.260	3.639	27.118	31.017	5.232	5.413	4.731	15.376	6.675	21.443
Q2	III	1.187	4.794	1.000	1.000	14.013	14.013	6.284	34.310	7.487	7.487	1.187	16.161	11.78	24.309

## Chapter 6

# Summary and Conclusions

### 6.1 Summary

This study presents the findings of an investigation of using different roughness coefficient formulation based on the different assumptions on the optimal design of composite channels. For formulation of models, seventeen different methods of calculation of equivalent roughness given by the past researchers of hydraulic engineering have been used. All these equations have their own limitations and assumptions. Using these equations in optimization problem, these limitations and assumptions modify the search space of the problem. Basic problem formulation is similar to the problem investigated by Das (2000) and Jain et al. (2004). These models have been solved for optimal design of trapezoidal composite channel using genetic algorithm (GA) and with classical optimization technique or sequential quadratic programming (SQP) from MATLAB 7.01 optimization toolbox. Three methodologies have been applied to investigate the impact of roughness on the optimal design of composite channels. The first methodology is to calculate equivalent roughness by dividing the total area into sub areas using imaginary vertical lines from the places of change in geometry or roughness, and the second methodology is to calculate the equivalent roughness bisection and the third methodology

is based on calculation of wetted perimeter along the imaginary vertical line. For this purpose two approaches has been used. The first approach is based on Poesy's (1967) consideration in which, wetted perimeter along vertical imaginary line was calculated for middle portion only and completely ignored in the others and second is based on the assumption that wetted perimeter along vertical imaginary line should be calculated for all subsections. For these approaches also the optimization problem has been solved for all the scenarios. For first two methodologies all the models have been solved for the optimal design of composite trapezoidal channel. For all the models and these two methodologies, two more scenarios have been investigated involving restrictions on top width and side slopes that may be warranted in the field, such as limited right of way, side slope stability criteria etc.

In an attempt to search for global solutions GA was applied to 20 different initial random populations for all the problems in this study. Though the global optimal solution can never be guaranteed, it was found that most of the search corresponding to a different initial random population of solutions resulted in similar optimal solutions indicating that the global optimal solution was probably reached. It was found that the results from GA for all the models and all the scenarios are comparable to those obtained by classical methods that also validate these results.

## 6.2 Conclusions

In this study, it was found that method of calculation of equivalent roughness can impact channel design considerably. The results from different modes having higher values of cost may not be correct. Model Q performed in the best manner amongst all the models for two methods and all the scenarios. Solutions by this model for Method-1 had 30.96% less cost than average cost of all the models and 34.27% less cross sectional area than the average cross sectional area of all the models. In case of Method-2, almost all the models had comparable cost but Model had least cost that was 2.66% lower than the average value of all the models. Also, in this case cost of all the models except models E to model H increased that showed that the results from this method are inferior to that of Method-1. In case of Scenario II and Scenario III also the Model Q had least cost and cross sectional area. In Method-1 Scenario II it had 31.58% lesser cost and 34.84% lesser area than the average value of the all models. Similarly, in Method-1 Scenario III it had 11.20% lesser cost than the average cost of all the models. In case of Method-2, for Scenario II and Scenario III this model had least cost and cross sectional area in all models. In the proposed methodology Model Q1 and Model Q2 had comparable cost in all scenarios, but in Model Q2 wetted perimeter along vertical imaginary lines was calculated in all segments that is a better approach and is recommended for optimal design of composite channel.

Each method is unique with unique assumptions. The method should be selected with proper care given to the site conditions in terms of availability of raw material in local



areas and appropriate assumption for site should be identified. Then the method most close to the assumption should be selected.

### **6.3 Future scope**

There are some another methods of division of area into sub areas. These methods can also be investigated. This study can be applied to the design of composite channels of different shapes like parabolic, triangular, semi circular etc. This study can also be used in the field problems having a main channel and a flood plain those have different roughness. In solving the optimization problem, for its practicability velocity constraint may be added.

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# Appendix-A

## Calculation of area of sub sections in bisecting line method

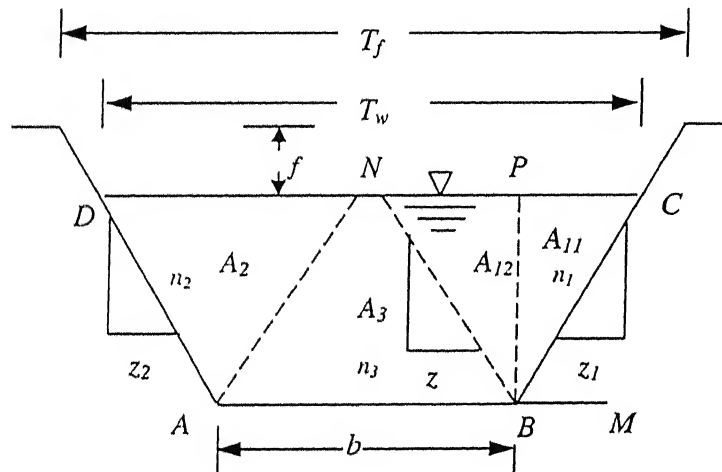


Figure A.1: Calculation of area of sub section

Let  $ABCD$  is flow area of the trapezoidal section. Let line  $BP$  is perpendicular to line  $DC$  and line  $BN$  is the line that is bisecting the angle  $\angle ABC$ . Let  $\angle CBM = \theta$ , hence the angle  $\angle ABN$  will be  $(90 - \theta/2)$  as line  $BN$  is bisecting the angle  $\angle ABC$ . From the above Figure in triangle  $\triangle BCP$

$$\tan \theta = \frac{1}{z_1} \quad (A.1)$$

Similarly in triangle  $\triangle BPN$

$$\tan (90 - \theta/2) = \frac{1}{z} = \cot \theta/2$$

$$\text{Hence, } \tan \theta/2 = z \quad (A.2)$$

From trigonometry

$$\tan \theta = \frac{2 \tan \theta/2}{1 - \tan^2 \theta/2} \quad (A.3)$$

Putting values of these two in equation (3)

$$\frac{1}{z_1} = \frac{2z}{1-z^2}$$

$$2z z_1 = 1 - z^2 \quad (\text{A.4})$$

it is a quadratic equation. Solving it we get

$$z = -z_1 \pm \sqrt{z_1^2 + 1} \quad (\text{A.5})$$

Area of the sub section  $\Delta BCN$  will be sum of area of  $\Delta BCP$  and  $\Delta BPN$ . Hence it will be

$$\frac{1}{2} (z_1 y) y + \frac{1}{2} (z y) y = \pm \sqrt{z_1^2 + 1} \frac{y^2}{2} \quad (\text{A.6})$$

Hence, the area of the subsection  $A_1$  will be

$$\sqrt{z_1^2 + 1} \frac{y^2}{2} \quad (\text{A.7})$$

Similarly, the area of  $A_2$  can be calculated. It will be

$$\sqrt{z_2^2 + 1} \frac{y^2}{2} \quad (\text{A.8})$$